COGNITION AND INSTRUCTION, *23*(4), 427–466

Inventing Mapping: Creating Cultural Forms to Solve Collective Problems

Noel Enyedy
Graduate School of Education & Information Studies
University of California at Los Angeles

In this article I detail the conceptual trajectory of a classroom of 2nd- and 3rd-grade students as they reinvent topographical lines to represent height in a map within the constraints of an overhead perspective. In my analysis I pay special attention to the role of social interaction—and in particular the role of the teacher—in the process of knowledge production. First, I demonstrate how the invention of representational forms by individuals occur as part of a larger social process of creating cultural conventions and negotiating a taken-as-shared understanding of these new tools. Second, I show how gesture, as a part of the larger semiotic ecology for meaning making around representations, contributes to creation of understanding. Third, I make some preliminary proposals regarding the process of transforming personal inventions into cultural conventions. The analyses are intended to contribute to our field's growing understanding of young children's activity when inventing representations (i.e., metarepresentational competence), the mechanisms for learning within instructional activities based on the iterative refinement of these representations (i.e., progressive symbolization), and a rejection of the dichotomy between an individual's cognition and her participation within a cultural community.

Representation—the act of highlighting aspects of our experience and communicating them to others and ourselves—is one of the fundamental and generative activities that is at the heart of the human experience. Sketches, diagrams, symbols, and so on, are a durable trace of our activity and thought that allow us to abstract, highlight, and coordinate salient aspects of the world around us. In doing so they shape what we and others see and remember about that experience. Though much is known about how students learn how to use various representational forms in

Requests for reprints should be sent to Noel Enyedy, University of California at Los Angeles, Graduate School of Education & Information Studies, 2323 Moore Hall, Box 951521, Los Angeles, CA 90095. E-mail: enyedy@gseis.ucla.edu

particular disciplines, the study of metarepresentational competence—what students understand about the process of representation itself (diSessa, Hammer, Sherin, & Kolpakowski, 1991; diSessa & Sherin, 2000; Nemirovsky & Tierney, 2001; Sherin, 2000)—and progressive symbolization—how the process of progressively refining one's representation of some aspect of the world can contribute to a deeper understanding of a domain (Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Hall & Stevens, 1995; Lehrer & Pitchard, 2002; Lehrer & Schauble, 2002)—are both relatively new topics of study within the learning sciences.

Over a decade ago, diSessa et al. (1991) traced the conceptual trajectory of a sixth-grade classroom that ends with the students reinventing conventional graphing. Since that time there have been a number of other studies that have examined the issues of how students reinvent other types of conventional representations and the ways in which their initial representations can progressively become more formalized, abstract, and mathematical (Bower, Cobb, & McClain, 1999; diSessa, 2004; Gravemeijer et al., 2000; Lehrer & Pitchard, 2002; Lehrer & Schauble, 2002; Lehrer, Strom, & Confrey, 2002; Nemirovsky & Tierney, 2001; Sherin, 2000). This body of work[1] sets the stage for this article in that this project was designed to engage students in the same type of progressive symbolization and formalization activities—activities that are oriented toward students inventing representations and iteratively refining solutions to problems that the class has identified. In this case, the students are involved in inventing ways to represent large-scale spaces and in the end come to reinvent many of the common conventions of mapping.

Although examining the ways people understand and represent large-scale spaces is an important topic in its own right and has generated a significant amount of educational and psychological research (Gauvain, 1998; Liben & Downs, 1989; Piaget, Inhelder, & Szeminska, 1960; Schofield & Kirby, 1994), the focus of this article is on the process of representational production and refinement and not on how well students learned to read or use maps. For the purpose of this article, maps are the normative symbol system that is the endpoint of the progressive symbolization. Like any symbol system, whether it is a programming language or mathematical notation, maps are powerful ways of seeing and understanding the world around us. The focus of this article is limited to the processes that students undergo as they learn to create, critique, and communicate with symbol systems. I argue that these types of metarepresentational competencies are closely related to the skills of abstraction and generalization, which are at the heart of science and mathematics. Understanding and learning how to foster metarepresentational competence is an important area of educational research.

---

[1]Although there are some differences in analytic focus between metarepresentational competence and progressive symbolization, I will refer to both types of studies under the more general label of progressive symbolization.

This article focuses on one particular representational invention that occurred midway through a unit on mapping the desert environment. A few days into the unit the students come to (re)invent topographical lines as a way to represent the height of an object mapped from an overhead view. My analyses trace the development of ways of reasoning linked to ways of representing space and highlight the ways in which an ecology of sign systems and meaning is built up around these representations in social interaction. Of particular importance to my analyses is the role of gesture and the way that it helps establish taken-as-shared understandings of these representations.

Figure 1 shows the end product of this process—a second grade student's representation of a dome. DiSessa et al. (1991) described the reinvention of conventional graphing as representing "genuine and creative work and that their accomplishment warrants study as an exceptional example of student-directed learning" (p. 117). Though that description also fits this study, the analyses of this article examine creative work in the context of the students' participation within a classroom community.

In the remainder of the article I ground my approach with the literature on progressive symbolization and gesture and then go on to describe my data sources and methods. In my analyses I make three claims. First, I argue that invention does not begin or end with the act of creation, but what might be called creative leaps of individuals are shaped by (and at the same time shape) a social matrix. In this case, students' representational innovations occur as part of a larger social process of creating cultural conventions and negotiating a shared understanding of these new tools. Second, I analyze the way the students and the teacher assembled resources (e.g., talk, images, and gestures) to understand and solve a collective problem. Third, I investigate the process of transforming personal inventions into shared conventions. In this process, I attempt to keep in focus both the role of the social in-

FIGURE 1 A second grader's map of a dome at the end of the unit. Transcript conventions are as follows:
= = latching
[over laps
(.) pause
[1]marks beginning of a gesture
ALL CAPS marks emphasis or increased volume.

teraction and the roles that students, as individuals, play in the process of transforming ideas—including the way cultural norms or cultural tools are adapted and changed as individuals appropriate them. In describing this "interactional work," I hope to contribute to the field's growing understanding of the coconstitution and reconciliation of an individual, psychological perspective with the collective, sociocultural perspective.

PROGRESSIVE SYMBOLIZATION

Learning through progressive symbolization is based on the assumption that the iterative process of creating a representation, followed by redescription and refinement of the representation, can lead to increasingly more and more sophisticated understandings of the content domain being represented (Lehrer et al., 2002). The refinement of one's ideas that accompanies the refinement of external representations is thought to occur because, during acts of invention, critique, and revision of the external forms students are reflecting on what they know and how to communicate it. During the course of these reflections and interactions students construct a complex set of personally meaningful relations between the model and what the model is intended to represent (Lehrer & Pritchard, 2002). Lehrer and Pritchard also argued that an understanding of a conventional representational form is just the tip of the iceberg. To really know what these representations mean, one must understand how the representational form relates to an intricate web of ideas including what the problem is and what a solution looks like. Learning through progressive symbolization packages up the learner's experience with that web of ideas and relations such that one can flexibly use the conventional form, can apply it to novel situations, and can understand and critique when the form is appropriate or when some other representation might be more valuable.

The design and analysis of this study is informed by a previous study in which Lehrer and Pritchard (2002) detailed the progressive symbolization of maps by third-grade students. Like this mapping study, the students initially represented their playground with drawn pictures that attempted to depict the space. From this initial representation the students added an origin, specified the scale, addressed the issue of orientation by adding a compass, and ended with a representational system that was for all intents and purposes a conventional map. Additionally, in designing the activities of this study I also drew on the activities described in Azevedo (2000), in which students were asked to create and critique representations of model and real landscapes. His analysis details the representations the students created, the resources used in the construction of those representations (such as perspective drawing, color, etc.), and the criteria they used to judge the adequacy of their own and other people's spatial representations.

My study extends these two studies (Azevedo, 2000; Lehrer & Pritchard, 2002) by tying the trajectories of successive refinement to the social interactions and structures behind the progression. In positioning my analysis as an extension of these two studies and other studies that address progressive symbolization, I do not mean to imply that these studies have ignored the role of social interaction or the role that the teacher played in orchestrating the activity and discussions. For example, diSessa et al.'s (1991) seminal study pointed out the many ways that the teacher made "organizational moves" (such as keeping track of the goal), provided "conceptual focus" (e.g., by suggesting certain tasks), and "kept alive multiple, child-originated representations" (p. 155) by naming them and often cleaning them up.

In later work (Azevedo, 2000; diSessa & Sherin, 2000; Elby, 2000; Granados, 2000; Sherin, 2000), some of the teacher's contributions and social interactions have been the topic of further analysis. For example, in Granados's analysis of the emergence of intersubjectivity in activities where students are designing programming algorithms to represent geometric figures, he found that the teacher used different types of moves in different phases of the design activity—beginning with an abundance of moves that excluded information and bounded the context and ending with a higher density of moves that specified information necessary for the solution. The teacher's moves are analyzed in terms of the way they modify the information states of the students. In this article, I take an interactional perspective and closely examine the interactions between what individuals accomplish and cultural resources that they use to actually get work done.

In the case study to be presented, the students' activity was oriented toward solving problems identified by the group based on their prior attempts to make maps. However, not all obstacles the students encountered led to group problem solving. Only those obstacles the group decided were significant received their collective attention. For example, one of the emergent, negotiated goals that oriented progressive symbolization in this case was to rebuild cities (made of wooden blocks) that had been "destroyed" (i.e., cleaned up and put away) based on their drawings of the cities. Because of the way the maps were drawn, knowing how high to rebuild a building emerged as a potential difficulty. In many contexts this particular problem would not be a problem that mattered, and recovering the exact height of the previous building would never be mentioned. In this case, the students themselves generated the relevance of and problems with representing height and, because it mattered to them, they invested their time and effort in solving the problem and coming to a consensus about the best solution.

INTERSUBJECTIVITY, THE SEMIOTIC ECOLOGY, AND GESTURE

The arguments of this article rest on the assumption that individual and shared understandings are achieved through a process of constructing and coordinating a semiotic ecology. By semiotic ecology I mean an overlapping set of sign systems that mutually reinforce and inform one another (Goodwin, 2000). The semiotic ecology includes talk, text, and graphics, as well as gesture, body position, material environment, participation structures, task structures, and history. Each resource is potentially a source of information. For example, an upward sweeping gesture might convey some sense of increase. Additionally, each resource can be the stage for another resource. To continue this example, the sweeping gesture may be performed over a line on a white board. The white board provides a stage for the gesture; the meaning of both the gesture and the line are modified by the combination. It is in interaction that resources are made to inform one another to create a rich web of meaning.

A resource that plays a central role in my analysis is gesture. This may be in part because of the spatial nature of the tasks and content domain. Gesture has been found to be a particularly good resource for disambiguating spatial information (Goldin-Meadow, 2001). In my analysis, gestures are often found to be the glue that binds together other resources into a coherent whole (cf. Roth, 2001). The scientific study of gesture is still an emergent discipline, one with competing theories about its relevance to education. For example, there is controversy concerning whether gesture is primarily communicative or is primarily one part of the computational stage used during the articulation of one's thoughts. Although there has been a substantial amount of research that supports the latter view (Alibali & Goldin-Meadow, 1993; Crowder, 1996; Crowder & Newman, 1993; Goldin-Meadow, 1999; Hadar & Butterworth, 1997), the perspective I adopt hypothesizes that gesture plays a role in communication and therefore a role in the progressive refinement of representations. From this perspective, the meanings of utterances in a conversation are underdetermined and the semantic content of language itself is modified by gesture and vice versa (Goodwin & Goodwin, 2002; Roth & Lawless, 2002). Talk and gesture (as well as other interactional resources) are taken as a unified package, mutually reinforcing and modifying one another, and contributing directly to our ability to establish socially shared perspectives and meanings (Goodwin, 2003b; Kendon, 1996).

For example, evidence shows that listeners treat gestures as communicative whether or not they are intended by the speaker to be so. In a study by Goldin-Meadow, Wein, and Chang (1992), adults watched videotapes of children explaining mathematical ideas and were asked to assess the child's understanding. In making their evaluations the adults often used information conveyed only through gesture and not expressed verbally by the child. This implies that in real interactions—interactions where the participants actually interact with one another—it is likely that one speaker's gestures are used by the interlocutor to modify their response. This is also consistent with the findings on recipient design from conversational analysis (Goodwin & Goodwin, 1987). My analysis focuses on two aspects of gesture and how gestures—whether they are consciously designed or not—contribute to the construction of socially shared meanings by interacting with other resources. First, I examine how gesture is used to highlight aspects of the material contexts the speaker addresses by pointing to or framing objects with their hands (cf. Goodwin, 2003a). Second, my analyses examine the ways that gesture plays a role in creating conversational cohesion (Koschmann & LeBaron, 2002). In their analysis of medical school, Koschmann and LeBaron demonstrated how gestures used by one speaker and later reused by a second speaker created a continuity that helped establish and maintain a shared context for their talk across turns. Likewise, Kendon (1990) provided examples of listeners synchronizing their own body movements and gestures with those of the speaker as a display of intersubjectivity.

CLASSROOM CULTURE: PURPOSE, INVENTIONS, AND CONVENTIONS

In analyzing aspects of the wider social structures of the classroom community that impact the students activity and development, I draw on Lehrer and Pritchard's (2002) insight that representational innovation is driven by problems the students discover as they try to accomplish a shared endeavor. It is the representational problem that drives the meaningfulness and relevance of a representational solution. In their study of students making and using maps, when the compass was introduced to the students before they struggled with how to orient the map, it was ineffective, "North was just a place on the map" (p. 27). This resonates with the idea that learning can be effectively embedded within purposeful activities and that these activities are "opportunities for gaining firsthand, practical experience of tackling problems in the relevant domain so that there will be a perceived need for the theoretical constructs that provide a principled basis for understanding those problems and the solutions to them" (Wells, 2000, p. 70).

In this article I also argue that an important aspect of classrooms designed around progressive symbolization is when one person's invented representation becomes a cultural convention for the classroom. Critical to this transformation in these analyses are issues of authorship. In this case, it is a repositioning of authorship—from the intellectual property of one individual to the idea being seen as being coauthored—that facilitates the adoption of the idea by the community.

What is at issue is both the development of a dominant discourse within the classroom and the appropriation of that discourse by the individual students. Of particular importance to my analysis is the way the teacher uses her asymmetrical status within the community to reposition ideas presented by one student as the intellectual property of the classroom, coauthored by everyone. Other analyses of classroom discourse have demonstrated the important role the teacher's revoicing and repositioning of ideas can be (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; O'Connor & Michaels, 1996). In this case, I argue that the change in authorship contributes directly to an idea becoming part of the dominant discourse and being appropriated by the majority of students (cf. Strom, Kemeny, Lehrer, & Forman, 2001).

In general, my theoretical stance places creative acts of individuals, such as the invention of topographical lines, within the context of a community struggling to invent cultural tools to solve collective problems. As such, the case presented in this article is intended to be a case, in microcosm, of cultural development. For classrooms, this means understanding the ways that an individual's cognition and conceptual development cannot be thought of as separate from the classroom's norms for participation and sociomathematical norms for disciplinary talk, and vice-versa (Yackel & Cobb, 1996). Cultural change and individual development are only meaningful in relation to one another.

METHOD

Participants

The study takes place in a combined second- and third-grade classroom with 22 students and one teacher, Ms. Adis.[2] The students are evenly split between second and third grade (ages 7 to 9). The students are also fairly evenly distributed by gender with 12 girls and 10 boys. The ethnic and socioeconomic status demographics of the school and classroom roughly reflect the demographics of California. The school's demographics are 32% Hispanic, 7% African American, 11% Asian, and 46% White, and the study classroom consisted of 4 Hispanics, 3 African Americans, 3 Asians, 10 Caucasians, and 2 students who were identified as mixed or other.

Task Design

At the time of the study, Ms. Adis had been teaching elementary school for 4 years, 2 years of which were at the present school. The school's curriculum is structured around long-term, in-depth themes and projects. For Ms. Adis's class, the year's theme was the desert environment and the animals and people that live in the desert. The activities for the mapping unit were collaboratively designed by Ms. Adis and me as a series of activities that might enrich students' understanding of the desert and the fit between features of the desert environment and the animals that live there. At the conclusion of the mapping unit the students took a field trip to the desert where they took hikes though the environment and visited a zoo devoted

[2]The names of the teacher and all the students are pseudonyms.

to local species of plants and animals. At the end of the field trip each student chose a desert animal to research.

The mapping unit consisted of six major activities spanning a total of 7 days, for a total of approximately 11 hr (see Figure 2). However, the activities were staggered such that the mapping unit took about 1 month to complete. Each activity was designed to create a genuine problem for students that they needed to solve, with each solution bringing the students closer to a deep, conceptual understanding of large-scale spaces and the conventional ways they are represented. Consistent with Lehrer and Pritchard (2002), our instructional design was intended to introduce students to a quantitative understanding of space and its properties (e.g., distance, direction, etc.) through successive refinements of the students' own inscriptions representing these properties.

This article examines the activities that were part of the second and fourth tasks, Block City and Rebuilding Block City. These two activities spanned 2 days and lasted approximately 4 hr. The activities were designed to raise a tension between a previously invented convention for the students' representation of space, the "bird's-eye view" (BEV), and how to represent the height of an object. A secondary tension was also expected to arise between solutions that preserved realism and those invented representations that sacrificed realism for systematicity.

The events analyzed for this study are as follows. First, the students constructed and represented a city made out of wooden blocks. This first step had the dual purpose of allowing students to practice using the BEV in their representations of space and also to set the stage for the expected difficulty with representing height. At the end of the 1st day, the block cities were torn down and the blocks put away, leaving only the students' representations of them. The second step, problem formation, began a few days later with a discussion of the representations and plans for reconstructing the city. In the discussion, the teacher raised the question, "what would be hard about rebuilding the cities." The students discussed several difficulties, eventually raising the issue that you cannot tell how tall different objects are

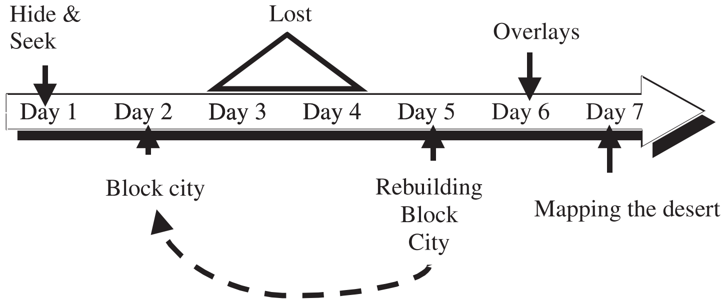


FIGURE 2 The series of tasks in the mapping unit.

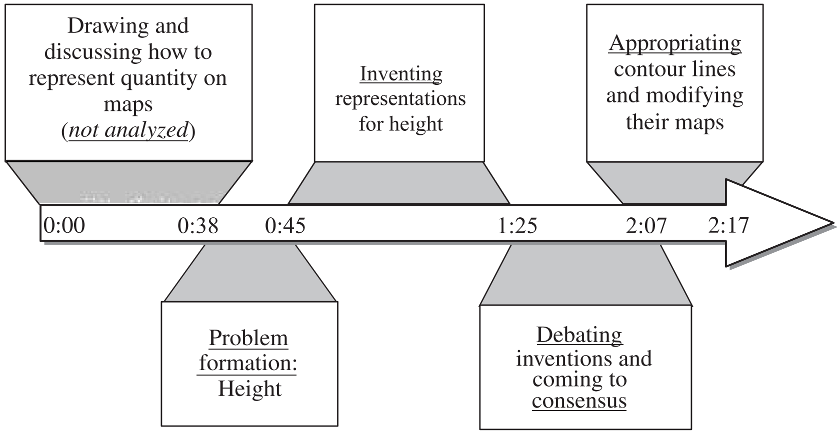


FIGURE 3 A timeline of the events of Day 5.

when they are drawn from the BEV. The third step involved inventing solutions for representing height on a set of shared geometric objects. The fourth step involved discussing and comparing the various representations invented by the students. This led to the fifth step, in which the students, as a group, came to a consensus about which way of representing height was the best solution. Finally, in the sixth step, students returned to "mapping" the geometric forms, this time in an attempt to use the solution on which they had agreed. The sequence of episodes that correspond to Steps 2 through 6 (i.e., problem formation through practice) and make up the bulk of my analysis are shown in Figure 3.

Sources of Data

I analyzed approximately 2 hr of videotape spanning 2 days that were 1 week apart. These two activities were chosen for analysis because they contained a critical advance in the ways students were reasoning about representing space. Toward the end of the 2nd day that is analyzed (Day 5), one student invents a way to represent height that closely resembles the conventional mapping convention of topographic lines (refer back to Figure 1). This was such a remarkable achievement that I knew even as I was videotaping that this event was worth investigating. Yet at the same time it occurred to me that, as remarkable as it was, it was also quite ordinary. This type of invention goes on all the time within any community engaged in purposeful activity.

My goal—to connect the stable ways students eventually come to represent a phenomenon to the moment-to-moment interactions when students are engaged in learning and conceptual change—drove my methods. From the discussion where topographic lines were introduced and adopted, my analysis goes "backwards" in time in an attempt to trace the origins of this invention and "forwards" in time to examine what subsequent impact it had on the way other students reasoned. The actual, practical work of analysis roughly follows the methods laid out by Hall (2001) and the methods of grounded theory (Glaser & Strauss, 1967). The videotapes themselves were first logged to outline the major events. All the tapes were then transcribed. Additionally, segments chosen for closer analysis were retranscribed to include the pauses, overlaps, intonations, and gestures that can often change the way an utterance is interpreted. The analysis progressed iteratively, sometimes working with the videos directly to better see the visual flow of the interaction, sometimes reading just the transcript to look for repeated words or patterns in phrasing that could easily be missed given the ephemeral nature of the spoken word. Finally, as often as possible I engaged in collective analyses of these episodes with my colleagues.[3]

In addition to the video record and the transcripts of the dialogue, the analysis examines the representations that the students produce on paper and the whiteboard during the activity. These inscriptions are evidence for conceptual growth (i.e., the products of the student learning) and data that speaks to how the students coordinate their own activity (i.e., they are critical to the process of learning).

RESULTS

The analysis of the series of activities that culminate in the class adopting the convention of topographic lines to represent height are presented chronologically. The first section describes the students' first attempts to map a space from the BEV. It is this experience, and the difficulties they discover, that established the context for the main analysis of this article. The second major section of the results section, analyzes the way the students and the teacher negotiate exactly what the problem is. The third analysis examines the students' representational inventions and how one of these representational forms was transformed into the classroom convention for representing height.

Exploration (Day 2)

In this phase of the activity the students worked in groups of four to build various "cities" out of wooden blocks (see Figure 4, left panel). The students were also engaged in representing their city with pen and paper (see Figure 4, right panel). Some students undertook these two activities concurrently, whereas other students first built the city and then made either a perspective drawing or a top-down draw-

---

[3]These collective viewings were sometimes formal workshops at conferences such as Datafest 2003 but, more often, informal sessions in my research groups.

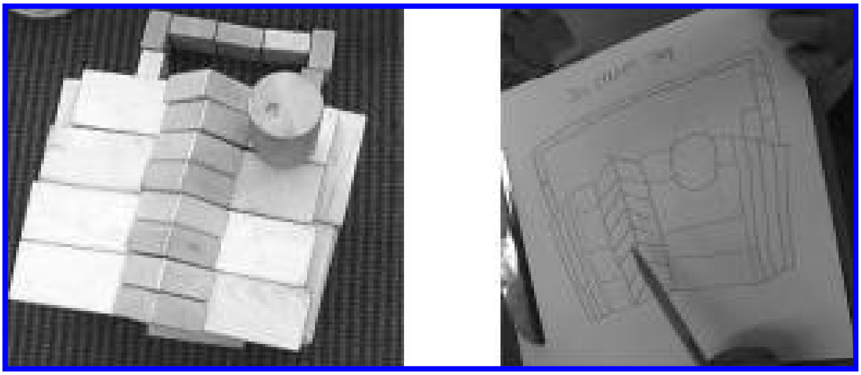


FIGURE 4 A part of a block city and its “map.”

ing they referred to as a map. At the end of the day, the students had a marvelous time pretending to become earthquakes, tornados, and other natural disasters that destroyed their cities, reducing their skyscrapers and freeways back into a pile of wooden blocks.

For both groups, the representational activity was focused on practicing the BEV—a top-down perspective—that was adopted by the class as a convention after a prior activity. Exactly how powerful a convention the BEV had become for some of the children became clear in an interaction between two group members who were trying to represent a freeway overpass. The first student drew a perspective drawing of the overpass that was recognizable as a picture of a bridge. However, a second group member took the map away from the first, erased the bridge, and redrew a rectangle in its place. When I asked why he erased his partner’s drawing he replied, “Um, because I looked at it again and I decided that you couldn’t see it from a bird’s eye view. See, when you look at that [points to the blocks] you couldn’t see that [points to what he just erased].” This exchange can be seen as a precursor to what will eventually become the problem with height. Although not framed as a problem with the a BEV’s ability to clearly represent the height of an object, this tension—between making the representation recognizable and systematically using the convention of a top-down view even when it make the object less recognizable—is at the heart of the students’ disagreement.

From this exchange several inferences can be made. First, the convention of BEV has not yet been appropriated by every member of the community. Some of the students still seem to be struggling with the tradeoff between systematically using a system for their maps and making their maps recognizable as the objects they represent. Whereas objects in a BEV map may still be recognizable, in certain cases salient aspects (e.g., height) that the students want to preserve are lost from this perspective. The struggle between the affordances and constraints of a representational system is a common event as new conventions are adopted by the community. From a design standpoint, this is why every activity was designed to lead to a new problem and was also seen as an opportunity for students to practice what they had learned previously. Occasions such as this disagreement often provide opportunities for students to teach one another the practices of the classroom (see Enyedy, 2003, for another example of this type of student-to-student teaching and learning moment).

Second, it seems clear that the BEV convention has been appropriated by some of the students. The student was quite forceful in the way he went about enforcing the convention, actually erasing his group-mate's work and replacing it with a "more appropriate" representation.

Third, it is worth noting that the concept of a top-down perspective has been codified by the term *bird's-eye view* within the classroom discourse. Although not all the students were using the representational convention yet, the technical term was recognized by everyone (or at the very least it was not questioned or challenged by students when it was used in conversation). The frequency with which this term was used and the lack of discussion about it during the mapping activity both support the conclusion that this phrase had become part of the taken-as-shared discourse practices of the classroom (Bowers, Cobb, & McClain, 1999). Overall, Day 2 was important because it provided the students with an experience that would eventually lead to a collective problem that they would solve by the addition of a new representational convention.

Negotiating the Problem (Day 5)[4]

There is a difference between what is problematic for an individual and what is a problem for the whole classroom. In the earlier exchange we saw that the unresolved problem of how to represent height in a map drawn from the BEV was implicit in a few instances. The question addressed in this section is as follows: How does this problem become the explicit, shared problem that will orient the whole community in a search for a solution? As others have pointed out, the anticipated purpose of a representation determines how one will design it and evaluate it (diSessa et al., 1991; Lehrer & Pritchard, 2002). Because this is a key event in the trajectory of the class, I analyze the 5 min when this is first discussed in close detail.

The Initial Problem

During the conversation in which the shared problem is established, the teacher plays several critical functions. Ms. Adis begins her whole class discussion of the

[4]For presentation purposes I refer to this as Day 5, but in the chronology of the classroom there were 7 intervening days between Day 2 and Day 5. In the week between Day 2 and Day 5 there was one 2-day mapping activity that was aimed at giving the students a chance to practice the BEV convention.

Block City activity by asking the simple question, "If it was your job to make—to rebuild—some of these cities based on these maps, what challenges would you have?" This question elicits several raised hands, and the teacher calls on Sarah, who says it is hard to see the details of the map because the students drew them so small (see Excerpt 1). In this case the gestures augment Sarah's talk to create visually a sense of just how small—so small you have to squint to see it. The gestures are shown in Excerpt 1 because they are part of the evidence that shows the transformation of ideas across students in the next several episodes.

Although the notion of scale is not directly relevant to the trouble the teacher wants the students to focus on, the students themselves are the ones who eventually transform Sarah's problem of scale into a problem about height. The first transformation is triggered by Alex (see Excerpt 2) who repeats parts of Sarah's talk and gestures and adds that knowing which block was being depicted in the map would also be difficult. The transformation is subtle but relevant to the joint work done to create a shared problem. Sarah's turn was focused on drawing the map and the difficulty of putting the details into such a small picture. Alex's turn focuses instead on the future activity of rebuilding the city and the use of the map to choose the correct blocks in the rebuilding process. Alex uses the same gesture as Sarah to link back to and establish continuity between turns while using his language to transform the idea. In a story about progressive symbolization, both aspects—continu-

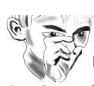
	Speaker	**Dialog**
1	Sarah	It will be really hard to show it because uh it is really small to see (.)
		How[1] (.) they drew it because it's really-it's like kinda smaller than my finger that I can do it.

*Holds fingers in a "C" shape and shrinks the space as she squints with one eye closed

		So it is really small to see how to put the detail inside[2]
		[2]puts one hand inside the cupped other hand
		and put the blocks together[3]
		[3]puts the heels of her hands together
2	Teacher	Okay, so absolutely the scale, we call that the scale of the map, right Sarah?
		That it is <u>drawn</u>[4] so small that it might make it difficult. Do people agree with Sarah about that?
		[4] motions hand toward the map held in her other hand

EXCERPT 1 Scale as a proposed problem for rebuilding the city (with relevant gesture drawn).

20	Teacher	What would make it difficult? And I need people back on the perimeter please. (8 sec)
21		What do you think?
22	Alex	I think it will be hard since it's so small[1] that it will be hard to know which piece was which.



Fingers fluctuating between big and small pinches

23	Teacher	Okay, Good, that was sort of Sarah's point too (.) that the scale made it difficult, right because it was so small. That is true, any other ideas about that?

EXCERPT 2 The gesture for scale is picked up while the idea is transformed to choosing the right blocks.

ity and transformation—are important. In this case, one result of the repetition of gesture is to maintain continuity and create conversational cohesion (Koschmann & LeBaron, 2002).

Using Their Shared History as a Resource

After discussing for several minutes the difficulties that scale could lead to in their representation, the teacher shows the class a student's map on which a rectangle is labeled *skyscraper*. In a sense, she is invoking a critical part of the students' shared history to "nudge" the conversation in the direction that she thinks will be productive. Here the "nudge" is an addition to the semiotic ecology.

Directly after viewing the map, Ms. Adis calls on Kevin, who begins a long narrative about the possible trouble they could have in rebuilding their city. His turn connects back to Alex's version of scale, where the problem was how to choose the right block, but changes the focus by raising the issue that, when using the BEV, some blocks can be occluded by blocks that are placed on top of them (see Excerpt 3).

24	Teacher	Kevin, what do you think?	
25	Kevin	Also, (.) because it's an overhead view and (.) it would be hard to (.) to do[1] the bottoms like say it was one thin piece (.) then one thin piece[2]. You wouldn't really[3] (.) by the size maybe by the size you wouldn't really actually (.) know[4] what size of block to take. Because there's different types of blocks. Big blocks[5], small blocks, rectangular blocks, triangle blocks, there's all kinds of blocks[6] and You have to know how to build the base[7] and up	[1] Places his hand horizontally with the palm down [2] Puts his other hand flat on top of the horizontal hand [3] Holding his pencil in his left hand he raises the hand then lets it fall and repeats the gesture up and down [4] Palms together are pulled apart horizontally [5] Makes a series of size gestures to go with each type of block [6] Places hand horizontally with palm up and then raises the hand
			
		and It's an overhead shot and it's an overhead view so basically all you see is what's on top and you don't see what's on the bottom[7].	[7] Puts hands together in a prayer/clapping motion

EXCERPT 3 Kevin transforms Alex's idea.

When Kevin is called on he begins by changing the focus of the class away from issues of scale and toward the problems that arise from taking a BEV. He builds on the notion of choosing blocks. He says, "because it's an overhead view and it would be hard to do the bottoms. Like say it was one thin piece, then one thin piece. You wouldn't really, by the size, actually know what size of block to take." As he talks about how one thin piece might be put on top of the other blocking, the view of the one on the bottom, he demonstrates what he means by placing one hand horizontally with the palm down and then placing his other hand over it, covering it up. To emphasize the vertical perspective that the map maker must take, he then takes his pencil and holds it vertically and begins to move it up and down. The tip of the pencil in effect becomes an arrow that shows the sight line of the observer, which is contrasted, with the stacks of blocks he had virtually created. He finishes his turn by saying, "there's all kinds of blocks, and you have to know how to build the base and up. And it's an overhead shot, so basically all you see is what's on top and you don't see what's on the bottom."

In this short student monologue Kevin is using several semiotic resources to slowly build a virtual world that will support his argument. He starts with the phrase, "because it's an overhead view and it would be hard to do the bottoms." This phrase establishes that there is a problem and locates it by verbally juxtaposing "overhead view" and "bottoms." However, imagine if that was the end of Kevin's turn. What would we understand at that point? I would argue that there is a significant amount of ambiguity at this point and a large potential for different interpretations of just what is "hard." Fortunately, Kevin continues and adds a new semiotic resource—a gesture—to further develop his intended meaning. The gesture of placing one hand on top of the other, combined with the phrase, "Like say it was one thin piece, then one thin piece," establishes a specific scenario that narrows down the possible interpretations of his turn. Kevin's next phrase, "You wouldn't really, by the size, actually know what size of block to take," links his turn back to the problem identified by Sarah and Alex in Excerpts 1 and 2. This brings in a third semiotic resource, the group's immediate history together, to help make sense of Kevin's turn. Finally, Kevin varies the potential size of the blocks to show exactly what the problem is. He takes the notion of a stack of think blocks and points out that there are "all kinds of blocks" and because "all you see is what's on top," you don't know what the bottom blocks look like. This is an illustration of what I mean by a semiotic ecology. Kevin weaves together three distinct semiotic resources, each one building on the other, and being understood in the context of the whole system.

Although the trouble with BEV representation of height is implicit in Kevin's problem it is unclear that Kevin or anyone else in the classroom understands his "problem" in that way. The teacher attempts to clarify Kevin's contribution by reflecting back what she heard and in doing so she invokes another piece of the class' shared history. In reiterating Kevin's turn, Ms. Adis uses her own body to reenact what many of the students had done when making their maps (see Excerpt 4).

30	Teacher	Let me see if I can say back to you what I think you said and see if this is right or not. I think what Kevin is saying[1] (1 sec) is that when you're looking from the bird's eye perspective[2], all we're seeing is[3] a certain perspective. We're seeing what we can from top[4] but we can't really tell what's on the bottom.	[1] Perches over the edge of her seat looking straight down with her right hand held next to her head [2] bounces hand up and down near her head [3] sweeps her hand horizontally with her palm down (see Figure 6) [4] drops her hand suddenly
31	Kevin	Sometimes (.) Sometimes there's some bottom sticking out[5]	[5] sweeps his hands out wide beyond his body
32	Teacher	Okay[6], am I understanding you correctly though? Am I understanding your idea?	[6] puts palm out to Kevin to halt his talk and holds it up during her turn
33	Kevin	Umm, Ya[7]	[7] nods head "yes"
34	Teacher	So sometimes you can see the bottom and sometimes you can't. Interesting. Does anyone want to respond to Kevin's idea? (3 sec)	

EXCERPT 4 The teacher reflects Kevin's idea back to him.

She begins by reflecting back to what Kevin had stated:

> Let me see if I can say back to you what I think you said and see if this is right or not. I think what Kevin is saying is that when you're looking from the bird's-eye perspective, all we're seeing is a certain perspective. We're seeing what we can from top but we can't really tell what's on the bottom.

As she says, "bird's-eye perspective," she bends over the edge of her seat (see Figure 5) to look straight down. This is exactly what many of the students had done when making their own maps—literally getting into a position where they can see the BEV of their cities (see Figure 5). Like the students, the teacher's turn takes full advantage of a range of semiotic resources in this exchange. Her phrase, "all we're

FIGURE 5 Ms. Adis uses her body to act out the bird's-eye view (line 30) and a student getting into bird's-eye view to make his map.

seeing is a certain perspective," is elaborated by her body position, which itself can be seen as an iconic link back to the children's prior history as a group.

Converging on a Shared Problem

Invoking this part of the class' shared history, a history that is strategically relevant to the topic at hand, leads to yet another transformation of the problem. To encourage the students to build on Kevin's idea, Ms. Adis takes a student map that includes a rectangle labeled *skyscraper* and redraws it and the label on the white board for everyone to see.

When the teacher says, "It's a skyscraper. This is the overhead view. What do you think about Kevin's point?" There is an immediate response. Alex, responds, "You wouldn't know how tall it could be. … It might be this tall or it might be this tall." Again the gestures that accompany Alex's talk contribute to how his talk is interpreted by the class. As he says, "it might be this tall or it might be this tall," he holds his hand next to the floor and then raises it 2 ft off the ground until his hand is level with the top of his head (see Excerpt 5 and Figure 6[5]).

His choice of words recreates the problem that he is claiming is in their maps. Because he uses the same deictic term, *this*, to refer to two different heights, the words do not disambiguate height. It is only when the talk is coordinated with the gestures that the difference in the two heights is made to be distinct.

This problem has immediate purchase for the community. In the left panel of Figure 7, I have highlighted the girl next to Alex as she performs the gesture with him during his second production of "It could be this tall or this tall." The timing of the two gestures can also be seen in the left-hand side of the previous figure, Figure 6. Likewise, in turn 60 (see Figure 7, center panel), Chris uses the same gesture when he says, "that looks like it is this tall." A few moments later another student reidentifies height as a problem using the same gesture, but with a new phrasing (Figure 7, right panel). I argue that repetition of the gesture is both evidence of and simultaneously contributing to the stabilization of height as the shared problem of the community.

Summary of the Problem Formation

I wish to make several analytical points relevant to my argument about the complexities of the social interactions that lead to successful instructional activities. The analyses of these excerpts are intended to elaborate the first two of my three main arguments. Namely, that the activity of the classroom community is driven by a semiotically relevant purpose produced and negotiated in interaction, and that

[5]The students and their parents have consented for their images to be used in publication. Some images are hand-drawn by the author to highlight the gesture and remove unnecessary visual information.

47	Teacher	Okay, let me show you one thing that's on the map. (1 sec) I'm just drawing it bigger[1].	[1] draws a square inside a square and labels it “skyscraper”
48	Teacher	It's a skyscraper. This is the overhead view. What do you think about Kevin's point?	
49	Alex Student 2	You wouldn't know how tall it could be. [it might be this tall	
50	Alex	[AND IT LOOKS FLAT	
51	Teacher	==Wait Alex	
52	Alex	==It might be this tall[2] or it might be this tall[3].	[2] hands held horizontal and close together [3] hands moved as far apart as possible
53	Teacher	Interesting, so you also don't know so not only so Kevin’s saying you don't know which box to use but Alex you're also saying you don't know how tall it is?	
54	Alex	[Right, it could be this tall[4]	[4] hand held two inches from the ground
55	Student 2	[It looks flat	
56	Kevin	[I said that also	
57	Student 2	[It looks like it's flat, look it	
58	Alex	=It[5] could be this tall or it could be this tall[6]. You wouldn't know[7].	[5] hand still poised two inches off the ground [6] hand moved to about two feet from the ground [7] hands spread apart in an opening gesture
59	Teacher	Interesting	
60	(Chris)	=That[8] looks like it's this [tall.	[8]Repeats Alex’s gesture with the hand first at the floor and then raised to eye level
61	Teacher	[That does make it look challenging to rebuild it. *((burst of student talk))*	
62	(Kevin)	=and that looks flat.	

EXCERPT 5 Creating the problem of height.

FIGURE 6 Alex shows *how* height is a problem using gestures.

FIGURE 7 The problem of height is appropriated by the class.

much of the conceptual work leading up to the invention of topographic lines is carried out collectively, with the teacher playing a strong role in connecting and elaborating a strong system of semiotic resources.

My first point about the series of interactions presented earlier and summarized in Figure 8 is that the formation of a shared problem that will shape their inventions and activity is produced socially and distributed across these exchanges. The connections between the ideas are not just objectively present, but are made and made relevant by the participants. In the series of excerpts shown (Excerpts 1 through 5), the critical path in the development of a shared problem was from (a) Sarah's idea of scale making it difficult to choose the right block, to (b) Kevin's idea that the occlusion of one block by another placed on top of it makes choosing the right block

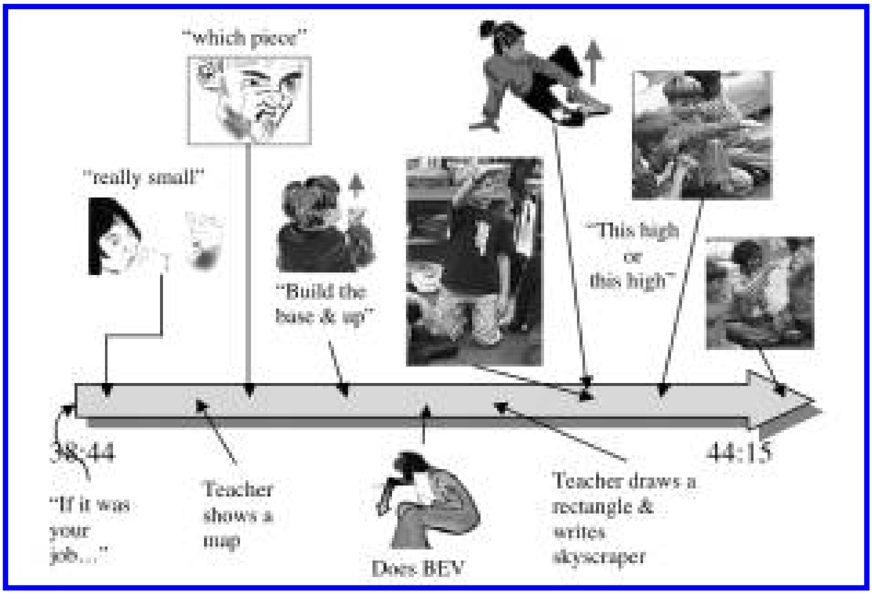


FIGURE 8 Elements and timing of the semiotic ecology.

difficult, to (c) Alex's idea that you can't tell how tall the object is from a BEV map, to (d) the appropriation of the problem by other students. Though Kevin's transformation builds off the idea of choosing and introduces the notion of stacking blocks and height, Alex's transformation builds off the idea of height and erases the problem of picking the right block.

Second, it is important to look at how these ideas, put in play by individuals, came to have a shared meaning. My analyses focus on the way language, gesture, and the material environment were woven together into a coherent semiotic ecology that contributes to a shared understanding of the problem even as the problem itself evolves. Each communicative resource both conveys information and sets the stage for interpreting the others (Goodwin, 1995).

A particularly striking resource that was used to create a shared meaning of the problem was gesture. By gesture I do not just mean movements of the hand, but the way in which the whole body is used during communication. For example, in Sarah's discussion of scale (Excerpt 1), she not only uses her hand in a pinching motion to create a virtually tiny object, but at the same time she closes one eye and squints the other, demonstrating just how hard it is to see the virtual object. What these interactions demonstrate is that the people interacting together use gesture as a resource to construct the meaning of the exchange. In this semiotic ecology, gesture was important because it

- Added nonlinguistic information to the context (e.g., showing a map).
- Added redundancy (e.g., saying "small" and showing small with a pinch).
- Modified other resources (e.g., saying "this tall" and showing how tall).
- Bound resources together (e.g., the teachers perch that enacted the BEV perspective to bring their shared history into the present) directing attention to particular parts of the shared environment (both material and linguistic).

Of particular importance here is the way in which one student's gestures help create the domain of scrutiny for the next student's contribution—a particular location where the addressee should look to find the relevant context for the interaction (Goodwin, 2003a). In this way the gesture connects one turn of the interaction to the next. For example, in Kevin's turn he establishes the domain of scrutiny to be a set of virtual blocks stacked on top of one another so that, when viewed from the top, the ones on the bottom are no longer visually available. He uses his pencil as a block and looks down on the pencil such that you can only see the eraser—modeling for the observer what is lost when one takes a BEV. His gestures create a frame of reference (with a visual anchor) that can be entered into by others and further modified by them.

Ms. Adis takes this domain of scrutiny—what things look like from a BEV—and elaborates it. In her turn, she produces a much more elaborate modeling of how one is positioned when looking at something from a BEV. She leverages their shared history of building and drawing the Block City when enacting the BEV. Her body position and gesture recreate a domain of scrutiny—their past activity—that is no longer physically present but exists vicariously in the residue of their collective history.

In addition to contributing to the shared understanding of an idea even as it changes, gestures, when they are taken up but not modified, can be seen as contributing to (and evidence for) the emerging stability of the idea. At the end of Alex's turn, his phrase "this tall or this tall" and accompanying gesture quickly circulate around the classroom both on the official and unofficial floor. One possibility is that the stability of the meaning of referent is advanced with each regesturing in the same way technical jargon is established by a disciplinary community. The use of the same gesture to describe different but similar situations and perhaps accompanied each time by different linguistic descriptions creates a web of meaning for the community.

Finally, the teacher played a number of specific, important roles in constructing the ecology. For example, when the teacher enacted the BEV, she was in effect reaching back in time to change the meaning of their current activity and helping them to visualize the difficulty with height. In other cases, she did the work to connect the material representations and language and make them mutually constitutive in the production of meaning. For example, when the teacher walked around with the map that had the word *skyscraper* written on it (and verbally spoke the word), it was the juxtaposition of the flatness of the image and the common knowledge that skyscrapers are tall that highlighted the problem that the teacher wanted to discuss. Additionally, she performed a number of moves to build both interactional continuity (i.e., explicitly referring to previous turns of talk) and topical continuity (building on other people's ideas).

The third and final analysis of this set of interactions has to do more generally with the teacher's role in orchestrating the discussion such that the right ideas are introduced, picked up, and "snowballed" (Anderson et al., 2001). The teacher in this case exhibited a balance between patience for the students to introduce and develop their own ideas and the ways in which she directed the conversation in the ways she wanted it to go. Perhaps one of the most difficult aspects of developing a constructivist classroom is to trust the students to voice their own ideas, invent solutions, and develop those ideas into something that approximates the intended curriculum. In this case, Ms. Adis exhibited her commitment to following the students' ideas and thinking in her exchange with Kevin. The problem with BEV and height was implicit in Kevin's contribution—it is a small step to go from not knowing which block is occluded to not knowing how many blocks are occluded. The teacher did not transform Kevin's idea into the problem she wanted to pursue. Instead, she revoiced it (O'Connor & Michaels, 1996) and cleaned it up in such a way as to set the context for a student to make the implicit idea of height explicit to the class. This is not to imply that teachers must always wait for every idea to be introduced by the students. It does demonstrate that the ways in which the teacher animates (Goffman, 1974) and transforms a student's contributions has implications for the agency that the child perceives herself to have and ultimately influences the ownership of the idea (see Gutierrez & Stone, 2002, for an example of the ways a teacher can restrict student agency and student learning).

In this section, I have outlined the process that the students and teacher engaged in to identify, understand, and collectively adopt the problem of height. My analysis highlighted the distributed nature of this process that involved (a) multiple students building on each other's ideas, (b) multiple resources that were bound together in interaction to create a shared understanding of the problem, and (c) the multiple goals and roles that the teacher balanced as she patiently elicited the children's ideas at the same time she directed the conversation in a specific direction. All the subsequent interactions of the classroom are oriented toward this object that the community established for itself.

Representational Invention (Day 5)

With a shared understanding of the problem established, the teacher turned the task at hand toward having the students invent a solution. Instead of having the students actually rebuild their block cities she asked each student to map the same environment, which was a landscape made up of five geometric forms (a step-pyramid, a dome, a cone, a bowl, and a series of alternating high and low blocks). This change was designed for two purposes. First, the teacher and I wanted the children to engage in struggling to represent the same forms. This was both to make sure they all encountered the same set of problems height could cause for a representational system, and to make it so that their invented representations were comparable to one another. Second, the geometric forms themselves were chosen carefully to seed the intended solution (topographical lines) and to verify if the solution was being used systemically. For example, the step pyramid was used to seed the idea of topographical lines as a possible representation for height. As a matter of fact, every student's map included a series of concentric squares to represent the pyramid. The dome and cone were included to see if that same invented strategy would be used systematically. In fact, only one student, working by himself, used concentric shapes consistently to map the different shapes.

The students, as a group, came up with several strategies to represent height in their maps. Throughout the article I refer to these strategies as "inventions." Given my recordings, it is impossible to say with any certainty that the students invented these representational systems from scratch. It is quite likely that aspects of the representation had their origins from contexts outside the classroom (cf. Roth & Bowen, 1995). Still, within the classroom community the teacher and the students attribute ownership to individual students as they reinvent these representational systems for the class, and it is on this basis that I refer to them as inventions. Given this limitation, my analysis will not examine the microprocesses of invention but instead will outline the results of the students' 40 min of activity.

In the whole class discussion immediately following their small-group mapping activity, three strategies were introduced by the students and discussed at length. Each invention is discussed in the order it was introduced to the class.

Invention 1: Shadows

The first, and by far the most common, strategy the students employed to show height was to add "shadows" to their representations by making certain line segments heavier and darker (see Figure 9, left panel). This strategy built on students' prior knowledge of what objects look like and their perspective drawing skills. In effect this representation was a dichotomous variable—it showed that something was tall, but in its original form it did not quantify the height of the represented object. This flaw was in fact pointed out by one of the students, and it led to a modification of the representation where the length of the shadow corresponded to the height of the object. It is worth pointing out that conventional maps also often use shadows to mark mountains (see Figure 9, right panel). Despite its limitations, the appeal for these students, and presumably for map makers in general, is that the shadows method provides perceptual fidelity—the representation looked like the object in question and the shadows gave the illusion of a third dimension.

Invention 2: Cartwheels

The second invention to represent the height of an object was introduced by a second grader named Chris. During the activity Chris had worked with another

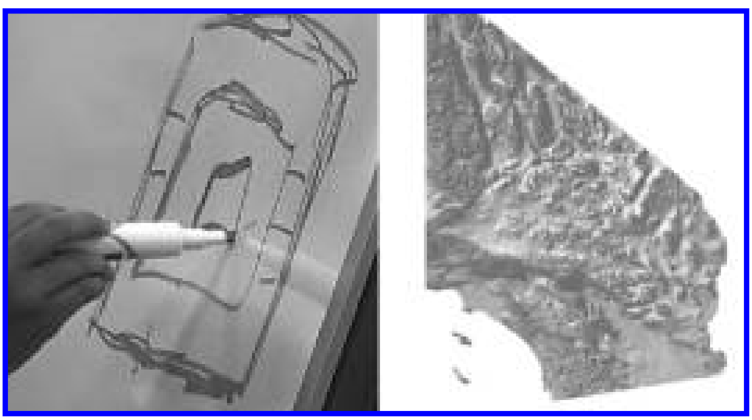


FIGURE 9 The step pyramid represented with shadows and a map of Southern California generated by a commercial Geographical Information System package.

student, Sherry, but during the public talk Chris takes the lead in explaining their idea. To keep the analysis clear, in the rest of the article I refer to Sherry and Chris's invention as Chris's idea. Chris went to the white board and drew a representation of the cone that consisted of a circle with spokes emanating from a point in the center, saying that their representation looked like a cartwheel. It was not obvious to Ms. Adis how the cartwheel represented height and so she asked Chris how the lines showed that the cone was tall. His reply was that you could use the distance between the shape that represents the base of the object and the point that represented the top of the object to show the height. Further, he argued that the distance of the line could be used to measure it (see Excerpt 6). The teacher then uses Chris's method to draw a tall and a short cone for the class (see Figure 10).

This is the first solution to show height quantitatively. What is interesting is that the representation sacrifices some of the perceptual fidelity to represent the height of the object systematically. Although this was not discussed by the children, to show the height of a very tall object one would have to distort the size of the base to

19	Teacher	Interesting. So how do those lines show us that it's tall?
20	Chris	Well, well, this is how far this is separated. And then how far the lines go. 'Cause if it was this tall, it'd look like that. It'll look a little taller. Like, it looks, I think it looks taller.
21	Teacher	So it's how far the lines go?
22	Chris	And how, how big the outer circle is.
23	Teacher	Oh, okay. So if the outer circle is bigger, it's taller? Is that your thinking?
24	Chris	Yeah, but it'll look taller.
25	Teacher	It looks taller if the outer circle is bigger. Okay. So this is Chris and Sherry's idea. All right. Another idea?

EXCERPT 6 The explanation of the Cartwheels representation.

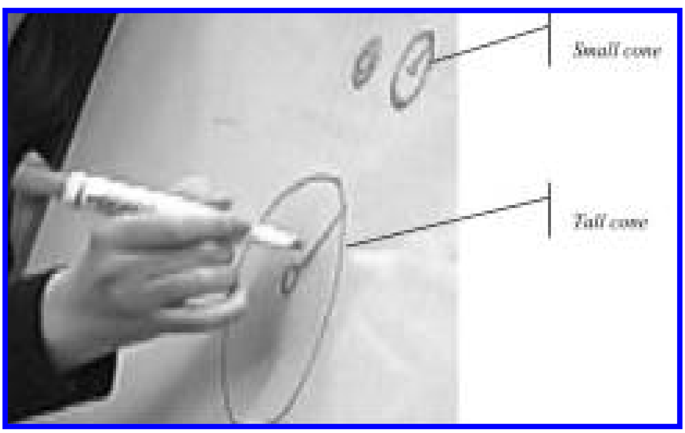


FIGURE 10 The teacher's reproduction of cartwheels with only one spoke.

create a long vertical line between the base and tip. A second flaw of this representation, again not discussed by the students themselves, was that the representation was incapable of representing the steepness of the object. However, in spite of these flaws, or perhaps because they were not discussed, this representation was greeted with some enthusiasm by the class.

Invention 3: Shapes Inside of Shapes (Topographic Lines)

The third invention for representing height was introduced by Matthew immediately following Chris's proposal for the Cartwheels method. Matthew and his partner Kim's invention was essentially a variation of topographic lines. To represent the height of the cone he used a series of concentric circles that ended with a point (see Figure 11). In Excerpt 7, Matthew introduces his idea in very simple terms.

1	Teacher	Any other ideas? Mathew, come on up by yourself. Kim, sit down on your chair	
	Matthew	We did it..we did it..a little like this. We made a big a circle, then made a smaller: Smaller:: smaller::: like that	
4	Teacher	Uh Huh <>	
5	Matthew	Then I did a tiny little circle	
6	Teacher	Okay, Mathew, would you turn around so other people can hear you. Explain why. Take a look at* Mathew's idea. Shhh.. Take a look at Mathew's idea.	*Teacher point to Mathew's illustration on the white board with her forefinger
7	Kim (off screen)	Kevin, don't, ( )	
8	Teacher	[Okay you ..Kim, focus in please. Mathew, explain your* idea	*Teacher audibly pats on the white board
9	Matthew	*I made a(.)made a big circle. I made a small one here and smaller and smaller till I got to this tiny circle	*Matthew moves closer to the white board and begins explaining by tracing his forefinger in an anti counter-clockwise direction, along the concentric circles, starting from the largest
10	Teacher	Okay, why did you do that?*	* Holding up the cone while asking the question
11	Matthew	We looked at it (like that) so it was going upwards*	* Right hand with fingers closed; arm makes a deliberate upward motion following the hand uphill

EXCERPT 7 Matthew presents his invention—Topographic lines.

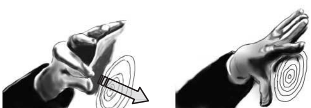
FIGURE 11 Matthew's representation of a cone.

Debate (Day 5)

There are now three proposals for representing height on the table, or in this case the whiteboard. The process of invention, at least in the context of a classroom oriented toward the production of knowledge, does not end with an act of creation. Debate contributes to the elaboration and clarification of an idea. Debate also can contribute to the stabilization and propagation of the ideas. Because this process of debate and consensus is another key point in the trajectory of the class, I again describe my analysis of the classroom interactions in detail.

In Excerpt 8 the teacher begins the debate by a restating Matthew's idea and adding a visual and gestural elaboration that forges the connection between the concentric circles and several slices of the cone. Chris, who had invented the Cartwheel method, immediately challenges the representation. In this case, the debate is facilitated by the fact that both Chris and Matthew have chosen to represent the same object, the cone. Having the same physical referent allows the students to more easily compare different aspects of the representation and perhaps to better see the flaws of the alternative representations.

In this exchange we can see that one result of Chris's critique is actually a further elaboration of Matthew's idea. The teacher's gestures both in line 12 and 19 of Excerpt 8 also help to clarify how the circles represent height by reinserting into the interactional space exactly what is missing from the two-dimensional representation—the physical height of the cone. In line 12, Ms. Adis positions her body in the bird's-eye perspective and uses her fingers to physically create circles that are fitted over different heights of the actual cone. On completion of the turn she points to the graphic display of concentric circles that Matthew had created. Gesture and body position are being used temporally to juxtapose a set of resources, which may help the students create a web meaning that binds these resources together into a whole. The body position invokes the classes shared history of looking down and drawing maps (as was discussed previously). The teacher's gesture of fitting circles over the cone is timed with her statement, "it kind of looks like a lot of little circles," binding together an element of the picture with an element of the physical space. Finally, the point at the end connects the whole turn to Matthew's representation.

12	Teacher	okay, hold on. [1] so Matthew's saying that it kinda looks like a [2] lot of little circles and that shows that it is going up? [3] 	[1] Teacher points Matthew to his seat [2] Teacher looks down the cone with the tip facing up, holding the cone with right hand and gripping the cone with left hand to show how the cone could be made up of a lot of little circles. [3] teacher points to Matthew's illustration and looks at Matthew
13	Chris	That looks like it's going down hill.0 0	
14	Teacher	Why do you say that Chris?	
15	Matthew	Looks to me like it's going up.	
16	Chris	Uh(.) uh it looks like it's [going down	
17	Kevin	[Different ways you look at it.	
18	Chris	[4]Looks like it's going up looks like some kind of tunnel, like one big circle, then you go further on and it get smaller=	[4] Chris walks up to the white board and flicks his fingers in front of the circle to illustrate the perception of a tunnel.
19	Teacher	=Ah, interesting. So from one perspective it could be the top and it's going down[5] but from Chris' perspective, it could be the top and it's going down this way[6]	[5] Teacher closes her thumb and forefingers and brings it to the above the center of the concentric circles and opens them in to out  [6] Teacher takes finger and moves them out and up and then back down to the surface of the whiteboard at the center of the circles 

EXCERPT 8 Debate about the merits of Matthew's topographic lines.

However, perhaps more important than the teacher's turn is the fact that Chris jumps in to challenge Matthew's strategy. Chris's challenge leads to further elaboration and clarification of Matthew's idea by the teacher and the need for it to be revised to make it unambiguous. As discussed later, it is these public challenges and subsequent revisions of Matthew's invention that begin to make it the class' shared convention.

My analysis of the debate phase of the classroom activity has focused on the joint construction and coordination of a semiotic ecology, anchored in this case by an invented representation. Similar to my analysis of the problem-finding activity, the semiotic ecology combined visual, linguistic, and social resources to establish what the representation represented and how it did its representational work. Through talk, inscriptions, and gestures the students and the teacher created a shared and elaborated understanding of Matthew's concentric circles representation.

Consensus (Day 5)

In Excerpt 9, the teacher orchestrates a critical shift in focus of the conversation. She shifts what they are talking about away from the presentation and elaboration of Matthew's idea to a discussion that facilitates the whole class' appropriation of Matthew's idea. The key move that Ms. Adis makes is to connect Matthew's idea to their collective history. She asks the class what in Matthew's invention is the same as what they all did when representing the step pyramid.

In this exchange, Ms. Adis juxtaposes elements of the semiotic ecology to create the right conditions to establish consensus and transform Matthew's invention into the class' convention. First, Ms. Adis calls the students' attention to the visual similarity between Matthew's representation of the cone and the shadows' representation of the step pyramid (refer back to Figure 9, left panel). Both these representations shared the same physical space on the whiteboard but they had not yet been publicly compared. Ms. Adis juxtaposes the two solutions by pointing to the step pyramid as she asks what the two representations have in common. In fact, she repeats the command to look for what they "see in common" twice in the same turn. As mentioned earlier, the step pyramid was included in the set of objects to be represented because, if the students attempted to maintain fidelity with their perceptions, they would be inclined to represent it as a series of concentric squares. This is indeed what did happen and what Ms. Adis was calling attention to. The effect of the spatial juxtaposition, highlighted by the teacher's talk and gestures, was to connect the two representations and make the similarity visible to the students. This leads Laurie to label the commonality for the class, which the teacher then marks as "interesting." The teacher then revoices the idea, attributing Laurie with

29	Teacher	What do you see in common between Matthew's idea and what a lot of you did on this? (1 sec) What do you see in common about those? Take a quick- take a good look. What do you see in common about Matthew's idea and everybody said they used this strategy for the pyramid. I need everybody's eye to be looking. Sit up. Sit up and put your face forward. Laurie, what do you notice?
30	Laurie	That there's a square inside a square inside a square and there it's a circle inside a circle inside circle=
31	Teacher	=Interesting, so this is one way that you think that we could show height is to put the shape inside of itself.
32	Student	And that looks kind of looks like a square tunnel hhh
33	Student	Yea it does!
34	Kevin	It could also look like a square peak

EXCERPT 9 The teacher moves the discussion toward consensus.

the belief that this is one way to "show height," and proceeds to write the name of the method on the board.

However, even more critical was the way the teacher changed the position and status of Matthew's idea. In line 29 of Excerpt 9, Matthew's idea is positioned as being something everyone had already done. The teacher, in effect, frames the visual connection between the two representations in terms of each student's personal history. Everybody had already used Matthew's idea at least once on the step pyramid. This is similar to what Lehrer et al. (2002) called "reaching back" (p. 393) to ground this conversation and productively push the group forward. I argue that in this case the reaching back pushes the group forward by subtly changing the ownership of the idea away from being Matthew's intellectual property and into the public domain. The change in status may be the critical move that creates the conditions for others to appropriate Matthew's. This move makes it so the students do not have to abandon their own strategy altogether and replace it with someone else's strategy. Instead, they are allowed to see appropriating Matthew's idea as modifying what they are already doing. However, it is worth pointing out that this move by the teacher does not end the debate and critique about the representations. At the end of Excerpt 9, the students are still debating if the concentric shapes could be confused with representing depth rather than height.

Another important move that contributed to the shift from invention to convention is the modification of Matthew's idea by others. In response to the ongoing debate regarding whether the "shapes inside of shapes" show height or depth, the teacher asks if the students could come up with a way to use color that would resolve the issue. After several false starts and failed solutions, a female student, Maya, suggests they add a key. Maya's key used different colored circles to correspond to different heights, introducing quantity directly into the representation to address the problem of height and depth. This resolved Chris's critique, because one could now refer to the key to see if it was a tunnel or a cone. The students had previously used keys on their maps to identify symbols used to represent specific types of items on the map (e.g., a tree), but keys had not yet been discussed as a potential solution to the problem with height. The modification of Matthew's idea by another student was yet another move that created a sense of shared ownership of the invention—once others had modified it, it had become theirs. I argue that this is a second move that helped establish concentric shapes as a convention.

The final move that helped to make topographic lines the class' cultural convention for representing height was when the teacher connected their invention to the professional practices of geographers. After Maya suggests adding a key, Ms. Adis says, "now you are thinking just like map makers," and proceeds to pull out a children's book about maps. She reads to the children several pages about how map makers used colored lines, called contour lines, to show the height of mountains and seafloors. This endorsement by the teacher (and by a published text) is the final move that stabilized Matthew's invention into the classes' shared convention.

Variations of these three moves were also used by the teacher when she established the only other strong convention for the classroom—the BEV. In this case, Ms. Adis connected the bird's-eye perspective to the students' personal histories of riding in airplanes and their shared history of trying to find hidden objects using perspective drawings on the 1st day of the unit. At one point she also polls the class to see how many think using the BEV is a good idea. I argue polling is a variation of the teacher's move that distributed authorship in that it gives each member of the classroom some agency in accepting or rejecting the idea. Finally, the teacher again related the convention of BEV to the professional practice of map makers. Though it is clear I must be cautious in drawing strong conclusions from two cases, the consistency in the moves made by the teacher is worth noting.

This is a different explanation than the strong cognitive position that explains conceptual change in terms of individuals making rational choices between competing ideas based on which idea is objectively better (e.g., Strike & Posner, 1992). Instead, the effect of the teacher's talk was to socially position the visual similarity of two representations as evidence that Matthew's idea was not unique. I argue it was the combination and mutual influences of the semiotic ecology and how it was socially positioned that created the conditions for the shift from invention to convention.

In this section I have outlined the process this classroom engaged in to move from debate to consensus. As was the case in my analyses in the previous sections, the material and social resources played an important role in this process. What was unique about this phase of the activity was the importance of how the ideas were positioned socially regarding other students' ideas. The teacher repositioned Matthew's idea as something that everyone had coauthored. The students then take up this invitation and begin to take over the ownership of Matthew's idea and modify it by adding a key. I argue this joint elaboration contributes to the snowball effect (Anderson et al., 2001) and helps to solidify this representation as *the* representation. The final act that contributed to the success of the representation was also social. It seems likely that the legitimacy of the representation was sealed when the teacher endorsed it as how map makers think and produced a published text as evidence. Perhaps any one of these social moves would have been enough to transform the invention to a convention, but together they were clearly a powerful combination.

Appropriation and Adaptation of a New Representational Practice (Day 5)

The teacher's reading of the map book is followed by her suggestion that they all go back to the geometric forms and try out using contour lines to represent at least one object. I argue that it is in this final aspect of the activity that we have strong evidence that the students have learned how to represent height at a deep conceptual level and are not just imitating Matthew or the standard conventions of topography. If students had only learned topographic lines by rote or at a surface level, one would expect that, when they used the convention, they would run into trouble, duplicate the convention exactly, or modify the convention in idiosyncratic and problematic ways.

However, when we examine the students' maps of the geometric forms following the debate, we see neither one consistent use of topographic lines nor any nonsensical variations of the convention. Instead, all the students successfully used topographic line to represent height. Further, we see three meaningful adaptations of the convention (see Figure 12). The first adaptation I have already noted, that is, Maya's introduction of a key. All of the students used a key in their final maps. There were two variations in the way that the students used the key. Some of the students used color in a qualitative manner—designating different colors to represent qualitatively different heights (e.g., "top, middle, and bottom"). Other students used the color to represent distinct quantified heights (e.g., "3 inches, 6 inches, and 9 inches"). Sometimes the heights were in consistent intervals, but sometimes the heights were chosen to match the actual heights of the objects—for example the second step of the step pyramid was an inch more that the other two steps. The students' third variation in the use of topographic lines was to make each line a specific and constant increase in height (e.g., 1 in.), thus eliminating the need for different colors. In fact, this third adaptation matches the standard convention for topographical lines used by the U.S. Geographical Survey as well as other map makers.

All in all, the second- and third-grade students were fully engaged and on task with this activity for just over 2 hr straight on Day 5. This, in itself, is a testament to the power of organizing instruction around meaningful and purposeful activity. Further, based on the students' subsequent use and modification of bands of color to represent aspects of the desert environment such as temperature and humidity, the activity as a whole seems to have led most (if not all) students to a deep and flexible understanding of how to represent large-scale spaces. However, it is beyond the scope of this article to present the detailed evidence that supports this

FIGURE 12 Keys to maps representing the three major adaptations of Matthew's idea.

claim. More important, the activity may have contributed to the students' metarepresentational competence.

DISCUSSION

Throughout this article I have argued that, to understand the accomplishment of the individuals within this second- and third-grade classroom, we must address the ways that individual agency stands in relation to participation in a community. Further, we must address the ways that the development of the community stands in relation to the actions and contributions of individuals. The complex relation between individual and community development only unfolds over time and thus requires to look across multiple episodes to see patterns of participation that link one episode to the next and create a whole that is more meaningful than a string of events.

Taken together, the episodes presented earlier support three analytic points that contribute to our understanding of this relation between individual invention and collective convention within progressive symbolization. First, invention does not begin or end with the act of creation, but creative leaps of individuals occur as part of a larger social process of creating cultural conventions and negotiating a shared understanding of these new tools. In my analysis, I highlighted that creativity and invention are driven by a purpose and that the purpose is found and negotiated in social interaction. During the interaction, the students were drawn in by the purpose, and it is this purpose that then oriented their individual action and coordinated their activity with each other (cf. Engeström, 1987).

Second, I analyzed the way that the students and the teacher opportunistically assembled resources to understand and solve their collective problem. The analysis showed that the teacher plays a particularly important role in the process at several key points along the trajectory, but that the work of building up the semiotic ecology was distributed over several participants. Additionally, how the teacher and students used their bodies and movements to augment and modify other resources for interaction (i.e., talk and visual images) emerged as an important dimension of how meaning was established around representational forms. Together, talk, images, the physical environment, and gesture created a semiotic ecology where each resource stood in relation to and informed the others. It was this semiotic ecology, as a whole, that was used to establish shared understandings and create a shared solution to the community's collective problem.

Third, I have argued that the change from an invented representation to a cultural convention is not merely an objective process of selection, but also a social process of coauthorship and transformation. Given the creativity of multiple individuals, the group as a whole (if it was to act in a coordinated way) had to collectively narrow the field and appropriate only a few of the invented solutions; in doing so those solutions were elaborated and modified by the group. In this process, I again highlighted the teacher's role in orchestrating the discussions that both created the desire to have a convention and transformed multiple, personal inventions into a convention. I also attempted to keep in focus the roles that students, as individuals, played in the process—including the way in which a cultural form was adapted and changed as individuals appropriated it.

Implications: Problems, Semiotic Ecologies, and Ownership

This case presents an alternative to the traditional ways of organizing learning in schools that may extend to other content domains. It shows how learning can be organized around students running into trouble, collectively coming together to understand the difficulty, adopting the problem as one they need to solve, and working individually and collectively to create a solution. The solution that they construct will likely have unforeseen entailments and start the whole cycle all over again—as was the case here. The trace of this iterative cycle is a progressively more refined and more symbolic representational system. The final form of this progression may be a single representational system that is appropriated by the members of the class, but because of the process the students went through, that final form is the tip of a web of understandings that were given meaning for each individual through their participation in collective activities. This perspective helps to explain why we did not just teach the students topographical lines at the beginning of the unit. First, solutions that are taught to students who do not understand the problem may very well be learned in a different manner, often at the expense of a deep understanding. If one were to teach the solution without taking the time to establish a shared understanding of the problem, many of the choices that were made in historical construction of what is now the canonical form would be hidden from the students. For example, without the tunnel-cone debate, the reason why conventional maps use color to depict height would remain a mystery to the students. It was the debate that made the necessity for the choice of using color or some other component of the representation visible and made the conventional solution meaningful. Second, competence at forming interesting and solvable problems is an important conceptual skill in its own right. However, the exact qualities of a question that can drive sustained, productive activity by a classroom is a topic that needs further research.

Designing classrooms organized around progressive symbolization also entails close attention to the conceptual tools and resources that students use to achieve their goals. In this case, I presented the set of resources that established a semiotic ecology for meaning making. Much of the analysis focused on how mundane aspects of interaction and everyday resources, such as gesture, connected one element of that ecology to another, making the whole more meaningful and providing a context where the conventional concepts and representations of the domain were understood at a deep level.

The use of pictures, gesture, and talk are by no means specific to conversations about math and science. However, that they are everyday resources makes them no less important for education. In fact, it could be argued that their "everydayness" and connection to informal activities make them more valuable. The increasing diversity in our classrooms and the perennial inequities of our schooling system create conditions where students come to the classroom with different discourse practices and different ways of making sense of the world; some of these ways are well matched with the requirements of schooling and others, though no less valid, are less valued in our current academic system. Making the magic of the mundane and everyday aspects of interaction transparent may help us better leverage the interactional resources that are common to students with different backgrounds and experiences so that they can better understand the formal and academic concepts and their canonical representational forms.

Finally, the analyses of this article are intended to help us as field better coordinate constructivist explanations of learning with sociocultural explanations. The constructivist perspective places analytic primacy on the ways that child learning is dependent on his or her prior knowledge and on the ways that they create meaning from their new experiences. The sociocultural perspective, though acknowledging that this is true, places its analytic primacy on the ways that the individual's experiences and the meaning they attribute to them are shaped by participation in culturally defined activities. Attending closely to the semiotic ecology is one way to avoid dichotomizing the individual and the social. In this case, my analysis shows that the semiotic ecology—which is tied directly to changes in the ways that individuals reason about and represent space—was the joint construction of multiple participants, involved multiple representational forms, and was coordinated via the participants' shared history. The meaning of a representation was often dependent on the participants' invoking their shared history together. For example, Ms. Adis leaned over the cone to reenact what the students themselves had done to experience a BEV and in doing so made the problem with height vicariously visible to the students. That a representation may only be meaningful in the context of the participants' history together highlights the social dimension to even material resources.

Other aspects of this case are far from mundane or everyday occurrences. Most prominent among these is the fact that this unit was designed. The changes to the students' cultural practices around mapping were planned. This is not typically how cultures develop. Saxe and Esmonde (in press), in descriptions of the ways that the Oksapmin people's quantitative practices have evolved over the last 20 years as a result of the introduction of currency, presented a much more common mechanism for cultural development. Saxe and Esmonde used an analogy with biological evolution to account for the changes in the way these people counted and performed arithmetic operations. In biological evolution, typically there is a change in the environment (such as a drought) that leads to a selective pressure that makes some features more desirable than others (e.g., longer beaks, which allow birds to better get seeds); over time these features become more prevalent in the population.

In the Oksapmin case, the introduction of currency into the culture was equivalent to an environmental change. The introduction of currency created new communicative contexts that needed to be solved, such as making change in a commercial transaction. These communicative contexts were analogous to a selective pressure. The selective pressure of making change eventually led to the invention of many idiosyncratic, quantitative practices that were local solutions to this problem.[6] Some of these solutions were more effective than others. Individuals, in their daily activities, repeated effective practices; as they interacted with a wider number of people, and as those people began to repeat the practices, some of those practices spread and became widely shared. Saxe and Esmonde proposed that this may have eventually led to a gradual change in the cultural practices without anyone being conscious of or in control of the process.

The cultural development described in this article is of a different sort. First, the teacher is actively constructing the selective pressures by the activities she introduces and her efforts to guide the discussion toward identifying particular problems to solve. Second, the process of this cultural development was different from the Saxe and Esmonde (in press) example. In the Oksapmin example, the process was a stochastic one, relying on large numbers of interactions and people's tendency to adopt practices that help them accomplish the goals or save them effort. In the mapping case, the process was both conscious and collective. It was conscious in the sense that it involved the intentional comparison of the merits and faults of the various inventions. It was collective in that the consensus process involved choosing one representational system that everyone agreed to try out.

In this case the selections of the problem and convention are by design determined by the teacher's curricular goals, which include her students' appropriating an existing cultural form. In this sense, the whole system is conservative. Though the ideo-culture of the classroom is developing, it is reconstructing the existing culture without substantially changing it. In the Oksapmin example, there was no design and no conscious process and as a result the culture developed in unpredictable ways. However, despite the appearance of being a conservative process of socialization, learning through progressive symbolization is not merely a transfer-of-knowledge model. Because the students are both discovering the task and the solution through their own activity, the process is a transformative one. The cultural meaning of the representational system of mapping was not just internal-

---

[6]Here the biological analogy breaks down slightly as these practices are invented by individuals, which is closer to the discredited Lamarckian theory of evolution than to that of natural selection.

ized, but was negotiated and appropriated in such a way that it could be used faithfully or modified in response to changes in context.

The conscious and reflexive process of coming to a consensus at the end of the progressive symbolization process merits further attention. The consensus phase requires students to, at least temporarily, put aside their personal preference (and perhaps the system that they themselves invented) to coordinate their activity with the other members of the classroom. Thus, in classrooms designed around progressive symbolization, student agency and intellectual ownership of ideas can come into tension with social coordination and group membership. In this case there was a potential tension for Chris, who had invented the Cartwheels representation and had identified a problem with Matthew's invention. The case also provides at least one way to navigate the tension. The teacher productively repositioned Matthew's invention as something that was owned by the group. She did this using three moves in the interaction. First, she invoked their shared history by helping the students see that everyone had already used Matthew's strategy. Second, this repositioning of the solution led the students to modify and elaborate Matthew's strategy, thus distributing the authorship and simultaneously moving the invention closer to the canonical form of contour lines. Third, the teacher connected their thinking process (and notably not their representation) to the disciplinary practices of map makers. It is worth noting that these three moves, which all involve distributing the authorship, has direct parallels with how some scholars have described the process of scientific discovery. For example, Latour (1987) argued that the removal of authorship is a part of scientific practice that contributes to a scientific theory's transformation into a scientific fact.

Although achieving consensus and conventionalizing invented representations were important to student learning in this case, they are topics that need further research. My conclusions are limited by the fact that there were only two cases of conventionalization in the unit. Though the teacher seemed to use similar moves in both cases, it is difficult to draw strong conclusions. Additionally, more research is needed to determine exactly when is the right time to push for consensus or to move to a conventional form. One can imagine that, if this move is made too early, individual students may be left behind or may adopt the solution without understanding it. Identifying the indicators of the right time to make these moves will be an important theoretical contribution with practical benefits for practice.

This article has described the events that led up to a genuine and creative accomplishment. The reinvention of topographic lines to represent height from a BEV is clear evidence of a high level of metarepresentational competence in this group of second and third graders. In analyzing how the activity worked, I have attempted to position the creative acts of individual students (as well as individual learning) within the social context of participating in a cultural group. At the same time, I have tried to show how the agency and actions of individuals influence and change that culture. In doing so, I hope to contribute to the ongoing efforts within the field of education to productively combine constructivist and sociocultural perspectives on learning and development and to demonstrate the way a classroom functions as a heterogeneous system of reasoning, including the coordination of talk, gesture, external representations, shared history, and shared goals.

ACKNOWLEDGMENTS

I thank Frederick Erickson, Mike Rose, Ray McDermott, Joshua Danish, and my two reviewers for their feedback on earlier drafts of this article. Their comments and suggestions improved both the depth of my analyses and the clarity of my writing. I also thank the participants of DataFest 2003 (Tim Koschmann, Richard Lehrer, Ricardo Nemirovsky, Gregory J. Kelly, Chris Fassnacht, Berthel Sutter, Bill Hanks, Curtis LeBaron, and Charles Goodwin) for their early feedback on my video analysis. Finally, this work would not have been possible without the collaboration of Sarah Wischnia.

REFERENCES

Alibali, M. W., & Goldin-Meadow, S. (1993). Transitions in learning: What the hands reveal about a child's state of mind. *Cognitive Psychology, 25,* 468–523.

Anderson, R. C., Nguyen-Jahiel, K., McNurlen, B., Archodidou, A., Kim, S., Reznitskaya, A., et al. (2001). The snowball phenomenon: Spread of ways of talking and ways of thinking across groups of children. *Cognition & Instruction, 19,* 1–46.

Azevedo, F. (2000). Designing representations of terrain: A study in meta-representational competence. *Journal of Mathematical Behavior, 19,* 443–480.

Bowers, J., Cobb, P., & McClain, K. (1999). The evolution of mathematical practices: A case study. *Cognition & Instruction, 17,* 25–64.

Crowder, E. (1996). Gestures at work in sense-making science talk. *Journal of the Learning Sciences, 5,* 173–208.

Crowder, E., & Newman, D. (1993). Telling what they know: The role of gesture and language in children's science explanations. *Pragmatics & Cognition, 1,* 341–376.

diSessa, A. A. (2004). Meta-representation: Native competence and targets for instruction. *Cognition and Instruction, 22,* 293–331.

diSessa, A. A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Meta-representational expertise in children. *Journal of Mathematical Behavior, 10,* 117–160.

diSessa, A. A., & Sherin, B. (2000). Meta-representation: An introduction. *Journal of Mathematical Behavior, 19,* 385–398.

Elby, A. (2000). What students' learning of representations tells us about constructivism. *Journal of Mathematical Behavior, 19,* 481–502.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki, Finland: Orienta-Konsultit.

Enyedy, N. (2003). Knowledge construction and collective practice: At the intersection of learning, talk, and social configurations in a computer-mediated mathematics classroom. *The Journal of the Learning Sciences, 12,* 361–408.

Forman, E., Larreamendy-Joerns, J., Stein, M., & Brown, C. (1998). "You're going to want to find out which and prove it": Collective argumentation in a mathematics classroom. *Learning and Instruction, 8,* 527–548.

Gauvain, M. (1998). Sociocultural and practical influences on spatial memory. In M. J. Intons-Adis & D. L. Best (Eds.), *Memory distortions and their prevention* (pp. 89–112). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research*. Chicago: Aldine.

Goffman, E. (1974). On face-work: An analysis of ritual elements in social interaction. In B. G. Blount (Ed.), *Language, culture and society: A book of readings* (pp. 224–249). Cambridge, MA: Winthrop.

Goldin-Meadow, S. (1999). The role of gesture in communication and thinking. *Trends in Cognitive Sciences, 3*(11), 419–429.

Goldin-Meadow, S. (2001). Giving the mind a hand. In J. McClelland & R. Siegler (Eds.), *Mechanisms of cognitive development: Behavioral and neural perspectives* (pp. 5–32). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Goldin-Meadow, S., Wein, D., & Chang, C. (1992). Assessing knowledge through gesture: Using children's hands to read their minds. *Cognition & Instruction, 9,* 201–219.

Goodwin, C. (1995). Seeing in depth. *Social Studies of Science, 25*(2), 237–274.

Goodwin, C. (2000). Practices of seeing, visual analysis: An ethnomethodological approach. In T. van Leeuwen & C. Jewitt (Eds.), *Handbook of visual analysis* (pp. 157–187). London: Sage.

Goodwin, C. (2003a). Pointing as situated practice. In S. Kita (Ed.), *Pointing: Where language, culture and cognition meet* (pp. 217–41). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Goodwin, C. (2003b). The semiotic body in its environment. In J. Coupland & R. Gwyn (Eds.), *Discourses of the body* (pp. 19–42). New York: Palgrave/Macmillan.

Goodwin, C., & Goodwin, M. (1987). Children's arguing. In S. Philips, S. Steele, & C. Tanz (Eds.), *Language, gender, and sex in comparative perspective* (pp. 200–248). Cambridge, MA: Cambridge University Press.

Goodwin, C., & Goodwin, M. (2002). Producing sense with nonsense syllables: Turn and sequence in the conversations of a man with severe aphasia. In B. Fox, C. Ford, & S. Thompson (Eds.), *The language of turn and sequence* (pp. 56–80). Oxford, England: Oxford University Press.

Granados, R. (2000). Constructing intersubjectivity in representational design activities. *Journal of Mathematical Behavior, 19,* 503–530.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modeling, and instructional design. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Communicating and symbolizing in mathematics classrooms: Perspectives on discourse, tools, and instructional design* (pp. 225–274). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Gutierrez, K., & Stone, L. (2002). Hypermediating literacy activity: How learning contexts get reorganized. In O. Saracho & B. Spodek (Eds.), *Literacy in early childhood education: Contemporary perspectives in early childhood education* (pp. 25–51). Greenwich, CT: Information Age.

Hadar, U., & Butterworth B. (1997). Iconic gestures, imagery, and word retrieval in speech. *Semiotica, 115*(1&2), 147–172.

Hall, R. (2001). Schedules of practical work for the analysis of case studies of learning and development. *Journal of the Learning Sciences, 10*(1&2), 203–222.

Hall, R., & Stevens, R. (1995). Making space: A comparison of mathematical work in school and professional design practices. In S. L. Star (Ed.), *The cultures of computing* (pp. 118–145). London: Blackwell.

Kendon, A. (1990). *Conducting interaction*. London: Cambridge University Press.

Kendon, A. (1996). Reflections on the study of gesture. *Visual Anthropology, 8,* 121–131.

Koschmann, T., & LeBaron, C. (2002). Learner articulation as interactional achievement: Studying the conversation of gesture. *Cognition & Instruction, 20,* 249–282.

Latour, B. (1987). *Science in action*. Cambridge, MA: Harvard University Press.

Lehrer, R., & Pritchard, C. (2002). Symbolizing space into being. In K. Gravemeijer, R. Lehrer, B. van Oers, & L. Verschaffel (Eds.), *Symbolization, modeling and tool use in mathematics education* (pp. 59–86). Dordrecht, The Netherlands: Kluwer.

Lehrer, R., & Schauble, L. (2002). Symbolic communication in mathematics and science: Co-constituting inscription and thought. In E. D. Amsel & J. Byrnes (Eds.), *Language, literacy, and cognitive development. The development and consequences of symbolic communication* (pp. 167–192). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Lehrer, R., Strom, D., & Confrey, J. (2002). Grounding metaphors and inscriptional resonance: Children's emerging understanding of mathematical similarity. *Cognition & Instruction, 20,* 359–398.

Liben, L. S., & Downs, R. M. (1989). Understanding maps as symbols: The development of map concepts in children. *Advances in Child Development and Behavior, 22,* 145–201.

Nemirovsky, R., & Tierney, C. (2001). Children creating ways to represent changing situations: On the development of homogeneous spaces. *Educational Studies in Mathematics, 45*(1–3), 67–102.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. In D. Hicks (Ed.), *Discourse, learning, and schooling* (pp. 63–103). New York: Cambridge University Press.

Piaget, J., Inhelder, B., & Szeminska, A. (1960). *The child's conception of geometry*. New York: Basic Books.

Roth, W.-M. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research, 71,* 365–392.

Roth, W.-M., & Bowen, G. M. (1995). Knowing and interacting: A study of culture, practices, and resources in a Grade 8 open-inquiry science classroom guided by a cognitive apprenticeship metaphor. *Cognition & Instruction, 13,* 73–128.

Roth, W.-M., & Lawless, D. (2002). When up is down and down is up: Body orientation, proximity and gestures as resources for listeners. *Language in Society, 31,* 1–28.

Saxe, G., & Esmonde, I. (in press). Studying cognition in flux: A historical treatment of 'Fu' in the shifting structure of Oksapmin mathematics. *Mind, Culture, and Activity.*

Schofield, N., & Kirby, J. (1994). Position location on topographical maps: Effects of task factors, training, and strategies. *Cognition & Instruction, 12,* 35–60.

Sherin, B. L. (2000). How students invent representations of motion a genetic account. *Journal of Mathematical Behavior, 19,* 399–441.

Strike, K. A., & Posner, G. J. (1992). A revisionist theory of conceptual change. In R. A. Duschl & R. J. Hamilton (Eds.), *Philosophy of science, cognitive psychology, and educational theory and practice* (pp. 147–176). Albany: State University of New York Press.

Strom, D., Kemeny, V., Lehrer, R., & Forman, E. (2001). Visualizing the emergent structure of children's mathematical argument. *Cognitive Science, 25,* 733–773.

Wells, G. (2000). Dialogic inquiry in education. In C. Lee & P. Smagorinsky (Eds.), *Vygotskian perspectives on literacy research* (pp. 51–84). Cambridge, MA: Cambridge University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.