Ed.231E Statistical Analysis with Latent Variables - Assignment 3

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Simple Growth Modeling

Q. *This assignment is open-ended and simply asks you to carry out growth modeling of the LSAY math achievement scores. You may use any subset of the data that you want and any covariates that you want.*

For this exercise, the Aggregate Math IRT scores (without aberrants: `mth7 - mth12`) were used as the repeated measures to be analyzed. The analysis process involves five majors steps, as summarized below:

Model 1: A linear growth model, without covariates.

Model 2: A non-linear growth model, without covariates.

Model 3: A non-linear growth model, with covariate "Home Math and Science Resources (`homeres`)."

Model 4: A non-linear growth model, with covariate "Mother's Education (`mothed`)."

Model 5: A non-linear growth model, with both covaraites "Home Math and Science Resources (`homeres`)" and "Mother's Education (`mothed`)."

Model 1

Expressing the repeated measures for each subject "i" at time "t" as the outcome measure (function of time t), the linear model that will be fitted for each individual subject can be expressed as follows:

$$\begin{aligned} y_{it} &= \eta_{0i} + \eta_{1i} \times t + \varepsilon_{it} \\ &= \mathbf{0} + \eta_{0i} \times \mathbf{1} + \eta_{1i} \times t + \varepsilon_{it} \qquad \text{where} \quad \varepsilon_{it} \sim N(0, \sigma^2) \end{aligned} \tag{1}$$

In a multi-level modeling framework, this would be referred to as the "Level-1" (within subject) model. And since we do not have any covariates in the model, the "Level-2" (between subject) model can be expressed as follows:

$$\begin{aligned} \eta_{0i} &= \alpha_0 + \zeta_{0i} \qquad \text{where} \quad \zeta_{0i} \sim N(0, \tau_{00}) \\ \eta_{1i} &= \alpha_1 + \zeta_{1i} \qquad \text{where} \quad \zeta_{1i} \sim N(0, \tau_{11}) \\ & \qquad \text{Cov}(\eta_{0i}, \eta_{1i}) = \tau_{01} \end{aligned} \tag{2}$$

When analyzing the repeated measures outcome in a structure equation modeling, is to treat Intercept (η_{0i}) and Slope (η_{1i}) as factors that are measured using the repeated measures as indicator variables. To properly

reflect this Factor Analytic view point in terms of mathematical expressions, equations (1) and (2) can be rewritten as follows:

$$\begin{aligned} y_{it} &= \boldsymbol{\nu} + \eta_{0i} \times \mathbf{1} + \eta_{1i} \times t + \varepsilon_{it} \qquad \text{where} \quad \nu = \alpha_0 \\ \eta_{0i} &= \mathbf{0} + \zeta_{0i} \\ \eta_{1i} &= \alpha_1 + \zeta_{1i} \end{aligned} \tag{3}$$

Rather than estimating the population intercept value at "Level-2," the value is now moved into the "Level-1" equation, thereby providing an intercept value for the Factor Analytic framework. Fig.1 illustrates this as a path diagram.

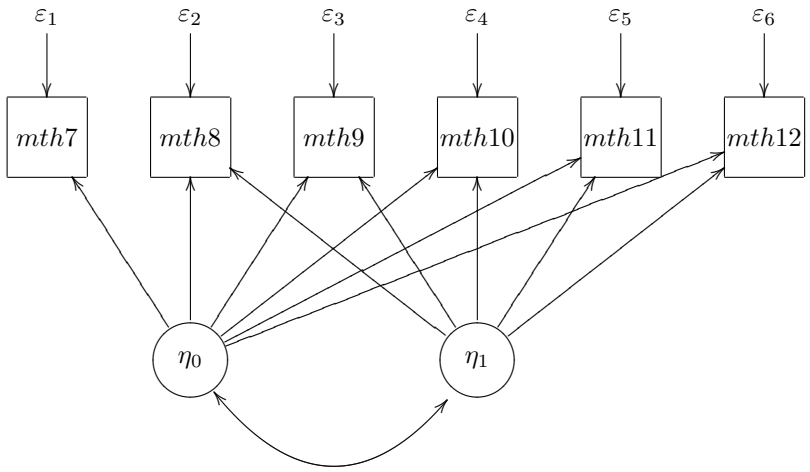

Figure 1: Path Diagram Illustrating Models 1 and 2

From equation set (3), it is clear that the factor loading of η_0 (the "intercept" factor: `MTHBETA0` in tables) will be **1** for all indicator variables (observed repeated measures), while the loadings of η_1 (the "slope" factor: `MTHBETA1` in tables) will be 0, 1, ..., 5 for `mth7`, `mth8`, ..., `mth12` respectively, since we are fitting a linear growth curve with equally spaced time-points.

The "`TYPE = basic`" analysis, or "`SAMPSTAT`" option under the `OUTPUT` command, in M*plus*, generates the variance-covariance and correlation matrices of all variables elected to be used for analysis. Table 1 is a correlation-covariance matrix of the 6 outcome measures that will be used for this analysis, which combines both matrices.

Table 1 indicates that Mean values of the Aggregate Math IRT scores appears to increase linearly. The variance of the repeated measures is monotonically increasing as well, which is typical of repeated measures data.

Table 2 summarizes the model fit statistics for current model under consideration, as well as all the models to be examined.

Table 1: Correlation-Covariance Matrix with Means

	MTH7	MTH8	MTH9	MTH10	MTH11	MTH12
MTH7	102.262	0.849	0.834	0.821	0.804	0.790
MTH8	93.060	117.362	0.886	0.863	0.846	0.827
MTH9	104.177	118.674	152.740	0.917	0.894	0.868
MTH10	109.689	123.525	149.643	174.532	0.928	0.896
MTH11	113.316	127.759	153.971	170.865	194.127	0.928
MTH12	118.731	133.243	159.573	176.032	192.296	221.093
Mean	53.472	57.243	62.184	66.690	69.374	70.290

The χ^2 tests is statistically significant ($p < .001$) indicating a poor fit. Similarly, the CFI[1] of 0.937 is below the recommended threshold value for good model fit. Furthermore, the TLI[2] of 0.941 is slightly below its threshold value for good fit. And finally, the RMSEA[3] of 0.188 is far above its threshold for goodness of fit.

Of the various fit indices for Model 1, SRMSR[4] at 0.057 is the only index where Model 1 clears the recommended theshold value.

The composite result of these model fit statistics suggest that Model 1 fits the data poorly. However, to be able to compare the results with those from the remaining models to be examined, the estimates and modification indices for Model 1 are summarized in Table 3 and Table 4 respectively.

Despite having the repeated measures observations `mth7`, `mth8`, ..., `mth12` as the indicator variables, the "Slope" Factor (`MTHBETA1`) appears with high modification indices with these observed variables. This signals that the strictly linear growth curve fitted under Model 1 may not be capturing the non-linear growth present in the data. This would justify the reason for relaxing this assumption in Model 2.

Model 2

The overall relationship between variables, first illustrated in fig. 1 remains the same for Model 2. The single difference between Model 1 and Model 2 is the factor loadings of η_1 (the "slope" factor) on `mth7`, `mth8`, ..., `mth12`. In Model 1, a linear growth curve with equally spaced time-points was fitted to the dataset, with factor loadings 0, 1, ..., 5. In model 2, however, the assumption of linear growth is relaxed, with factor loadings 0, 1, t_2, ..., t_5. This will allow the model to become more flexible and account for consecutive time-point growth to differ from one another.

Referring back to Table 2, the χ^2 tests remains statistically significant ($p < .001$) indicating a poor fit.

[1] A χ^2 comparison of the target model to the baseline model. It is generally recommended that the value be equal or greater than .96/.95.

[2] Another form of χ^2 comparison of the target model to the baseline model. It is generally recommended that the value be equal or greater than .95/.95.

[3] A function of χ^2 which also test model fit. It is generally recommended that the value be less than .05.

[4] The average residual for the correlation matrix. It is generally recommended that the value be less than or equal to .07/.08.

Table 2: Model Fit Statistics Summary

Growth Curve Covariate(s)	**Model 1** Linear -	**Model 2** Non-linear -	**Model 3** Non-linear homeres	**Model 4** Non-linear momed	**Model 5** Non-linear homeres, momed
Chi-Square Test of Model Fit					
Value	457.907	81.699	86.373	90.123	93.626
Degrees of Freedom	16	12	16	16	20
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000
Chi-Square Test of Model Fit for the Baseline Model					
Value	7056.973	7056.973	7003.030	6943.026	6851.155
Degrees of Freedom	15	15	21	21	27
P-Value	0.0000	0.0000	0.0000	0.0000	0.0000
CFI/TLI					
CFI	0.937	0.990	0.990	0.989	0.989
TLI	0.941	0.988	0.987	0.986	0.985
Loglikelihood					
H0 Value	-15183.165	-14995.061	-15904.606	-15573.417	-16434.400
H1 Value	-14954.212	-14954.212	-15861.420	-15528.356	-16387.586
Information Criteria					
Number of Free Parameters	11	15	17	17	19
Akaike (AIC)	30388.331	30020.123	31843.212	31180.835	32906.799
Bayesian (BIC)	30439.611	30090.051	31921.934	31259.579	32994.249
Sample-Size Adjusted BIC (n* = (n + 2) / 24)	30404.681	30042.418	31867.952	31205.596	32933.917
RMSEA (Root Mean Square Error Of Approximation)					
Estimate	0.188	0.086	0.076	0.078	0.071
90 Percent C.I.	[0.173 0.203]	[0.069 0.104]	[0.061 0.092]	[0.063 0.094]	[0.057 0.085]
Probability RMSEA $\leq$. 05	0.000	0.000	0.003	0.002	0.009
SRMR (Standardized Root Mean Square Residual)					
Value	0.057	0.016	0.015	0.015	0.014

Table 3: Model 1 Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
MTHBETA0 |					
MTH7	1.000	0.000	0.000	9.472	0.914
MTH8	1.000	0.000	0.000	9.472	0.859
MTH9	1.000	0.000	0.000	9.472	0.793
MTH10	1.000	0.000	0.000	9.472	0.730
MTH11	1.000	0.000	0.000	9.472	0.682
MTH12	1.000	0.000	0.000	9.472	0.602
MTHBETA1 |					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	1.000	0.000	0.000	1.547	0.140
MTH9	2.000	0.000	0.000	3.094	0.259
MTH10	3.000	0.000	0.000	4.640	0.358
MTH11	4.000	0.000	0.000	6.187	0.445
MTH12	5.000	0.000	0.000	7.734	0.492
MTHBETA1 WITH					
MTHBETA0	6.715	0.688	9.754	0.458	0.458
Means					
MTHBETA0	54.067	0.356	152.003	5.708	5.708
MTHBETA1	3.744	0.067	56.204	2.421	2.421
Intercepts					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	0.000	0.000	0.000	0.000	0.000
MTH9	0.000	0.000	0.000	0.000	0.000
MTH10	0.000	0.000	0.000	0.000	0.000
MTH11	0.000	0.000	0.000	0.000	0.000
MTH12	0.000	0.000	0.000	0.000	0.000
Variances					
MTHBETA0	89.727	5.020	17.874	1.000	1.000
MTHBETA1	2.393	0.180	13.273	1.000	1.000
Residual Variances					
MTH7	17.627	1.379	12.781	17.627	0.164
MTH8	16.140	1.061	15.213	16.140	0.133
MTH9	16.457	1.000	16.451	16.457	0.115
MTH10	16.740	1.041	16.085	16.740	0.099
MTH11	11.212	0.974	11.515	11.212	0.058
MTH12	30.660	2.062	14.867	30.660	0.124

Table 4: Model 1 Modification Indices

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
BY Statements				
MTHBETA0 BY MTH7	37.129	-0.024	-0.229	-0.022
MTHBETA0 BY MTH8	24.461	-0.016	-0.148	-0.013
MTHBETA0 BY MTH9	28.567	0.016	0.148	0.012
MTHBETA0 BY MTH10	115.875	0.032	0.301	0.023
MTHBETA0 BY MTH11	12.721	0.011	0.102	0.007
MTHBETA0 BY MTH12	240.982	-0.068	-0.644	-0.041
MTHBETA1 BY MTH7	30.500	-0.304	-0.471	-0.045
MTHBETA1 BY MTH8	31.797	-0.247	-0.382	-0.035
MTHBETA1 BY MTH9	26.550	0.208	0.322	0.027
MTHBETA1 BY MTH10	113.768	0.436	0.674	0.052
MTHBETA1 BY MTH11	22.100	0.199	0.308	0.022
MTHBETA1 BY MTH12	273.795	-1.011	-1.564	-0.099
WITH Statements				
MTH8 WITH MTH7	8.360	3.582	3.582	0.031
MTH9 WITH MTH7	13.448	-3.352	-3.352	-0.027
MTH9 WITH MTH8	9.021	2.385	2.385	0.018
MTH10 WITH MTH7	10.696	-2.784	-2.784	-0.021
MTH10 WITH MTH8	8.017	-2.199	-2.199	-0.015
MTH10 WITH MTH9	61.751	6.075	6.075	0.039
MTH11 WITH MTH8	16.166	-3.001	-3.001	-0.020
MTH11 WITH MTH9	7.137	-2.075	-2.075	-0.013
MTH11 WITH MTH10	18.266	3.851	3.851	0.021
MTH12 WITH MTH7	31.627	6.834	6.834	0.042
MTH12 WITH MTH8	9.119	3.176	3.176	0.018
MTH12 WITH MTH9	18.723	-4.553	-4.553	-0.024
MTH12 WITH MTH10	53.342	-8.505	-8.505	-0.042
MTH12 WITH MTH11	18.158	6.557	6.557	0.030

However, it is important to note that the χ^2 value has dropped dramatically, indicating a significant improvement in model fit. The CFI of 0.990 is above the recommended threshold value, indicating that the model fits the data well. The TLI of 0.988, which is also well above its threshold value, supports this as well. The same can be said for SRMSR, which at 0.016 has cleared its threshold value.

The RMSEA of 0.086 is still above its threshold for goodness of fit, but is significantly smaller when compared to the value for Model 1.

The composite result of these model fit statistics suggest that Model 2 fits the data well. The estimates and modification indices for Model 2 are summarized in Table 5 and Table 6 respectively.

In Table 5, we notice that the slope does change with the steepest growth observed from grade 8 to grade 9, followed by the second largest growth between grade 9 and 10. Interestingly, although all slopes from grade 9 on are significant (Est./S.E. > 1.96) the steepness of the slope appears to lessen with higher grade level. One may speculate that this is attributable to increased difficulty in the subject of math, thereby making it more difficult for students to maintain the same level of growth at upper grades.

In Table 6, we notice that the "Slope" factor (`MTHBETA1`) no longer appears, which indicates that the model now sufficiently accounts for non-linear growth in the data. The remaining modification indices related to the "Intercept" factor (`MTHBETA0`) have dropped in magnitude as well.

The R^2 for each of the repeated measures in the model is summarized in Table 7. As it can be seen from the table, the R^2 is higher under Model 2 than Model 1 for all manifest variables.

Table 8 summarizes the Factor Determinacy[5] for the "Intercept" (`MTHBETA0`) and "Slope" (`MTHBETA1`)factors across all models examined in this assignment. Both factors have higher values under Model 2 over Model 1.

Model 3

For the sake of the assignment, let us hypothesize that previous research have shown "Mother's level of education" and "Home Math and Science Resources" to be crucial covariates that influence a student's initial status on the Aggregate Math IRT scores at grade 7, as well as his/her growth rate. The aim of the following models is to evaluate this results based on the LSAY dataset.

We will begin by introducing one of the covariates, "Home Math and Science Resources" (`homeres`) into the model. To reflect this addition of time-invariant covariate in a mathematical notation, the equation set (3) will now be rewritten as follows:

$$\begin{aligned} y_{it} &= \boldsymbol{\nu} + \eta_{0i} \times \mathbf{1} + \eta_{1i} \times t + \varepsilon_{it} \qquad \text{where} \quad \nu = \alpha_0 \\ \eta_{0i} &= \mathbf{0} + \gamma_0 \times w_i + \zeta_{0i} \\ \eta_{1i} &= \alpha_1 + \gamma_1 \times w_i + \zeta_{1i} \end{aligned} \tag{4}$$

[5]Indicates how close the average estimate is to the true factor score. It is generally desirable to have a value of 0.80 or higher.

Table 5: Model 2 Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
MTHBETA0 |					
MTH7	1.000	0.000	0.000	9.401	0.920
MTH8	1.000	0.000	0.000	9.401	0.865
MTH9	1.000	0.000	0.000	9.401	0.777
MTH10	1.000	0.000	0.000	9.401	0.709
MTH11	1.000	0.000	0.000	9.401	0.671
MTH12	1.000	0.000	0.000	9.401	0.638
MTHBETA1 |					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	1.000	0.000	0.000	1.534	0.141
MTH9	2.369	0.106	22.343	3.634	0.300
MTH10	3.581	0.165	21.755	5.493	0.414
MTH11	4.338	0.202	21.437	6.655	0.475
MTH12	4.593	0.218	21.097	7.046	0.478
MTHBETA1 WITH					
MTHBETA0	6.084	0.706	8.618	0.422	0.422
Means					
MTHBETA0	53.527	0.364	147.002	5.694	5.694
MTHBETA1	3.659	0.198	18.506	2.385	2.385
Intercepts					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	0.000	0.000	0.000	0.000	0.000
MTH9	0.000	0.000	0.000	0.000	0.000
MTH10	0.000	0.000	0.000	0.000	0.000
MTH11	0.000	0.000	0.000	0.000	0.000
MTH12	0.000	0.000	0.000	0.000	0.000
Variances					
MTHBETA0	88.373	4.952	17.845	1.000	1.000
MTHBETA1	2.353	0.296	7.946	1.000	1.000
Residual Variances					
MTH7	15.995	1.359	11.771	15.995	0.153
MTH8	15.221	1.033	14.737	15.221	0.129
MTH9	16.033	0.960	16.696	16.033	0.109
MTH10	13.558	0.900	15.060	13.558	0.077
MTH11	11.104	0.942	11.788	11.104	0.056
MTH12	23.218	1.503	15.448	23.218	0.107

Table 6: Model 2 Modification Indices

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
BY Statements				
MTHBETA0 BY MTH7	16.450	-0.083	-0.779	-0.076
MTHBETA0 BY MTH8	4.519	0.031	0.296	0.027
MTHBETA0 BY MTH10	4.102	0.021	0.199	0.015
MTHBETA0 BY MTH11	7.275	-0.034	-0.316	-0.023
WITH Statements				
MTH9 WITH MTH7	7.954	-2.472	-2.472	-0.020
MTH9 WITH MTH8	19.561	3.363	3.363	0.026
MTH10 WITH MTH9	32.425	4.047	4.047	0.025
MTH11 WITH MTH8	4.939	-1.568	-1.568	-0.010
MTH11 WITH MTH9	15.575	-2.936	-2.936	-0.017
MTH12 WITH MTH7	7.619	2.751	2.751	0.018
MTH12 WITH MTH9	9.059	-2.669	-2.669	-0.015
MTH12 WITH MTH10	19.044	-4.292	-4.292	-0.022
MTH12 WITH MTH11	49.269	8.298	8.298	0.040

Table 7: R^2 by Model

	R^2				
Observed Variable	Model 1	Model 2	Model 3	Model 4	Model 5
MTH7	0.836	0.847	0.844	0.844	0.842
MTH8	0.867	0.871	0.872	0.871	0.872
MTH9	0.885	0.891	0.890	0.891	0.890
MTH10	0.901	0.923	0.923	0.923	0.923
MTH11	0.942	0.944	0.942	0.944	0.943
MTH12	0.876	0.893	0.892	0.892	0.892
Latent Variable					
MTHBETA0	-	-	0.186	0.128	0.245
MTHBETA1	-	-	0.097	0.027	0.107

$R^2 = 1 - \text{STDYX} = \text{Reliability}.$

Table 8: Factor Determinacies by Model

	Model 1	Model 2	Model 3	Model 4	Model 5
MTHBETA0	0.968	0.967	0.967	0.967	0.967
MTHBETA1	0.870	0.881	0.880	0.882	0.881

From a multi-level framework, w_i is a "between subject" difference, the covariate is included at Level-2. From a structure equation modeling framework, the "Intercept" and "Slope" factors now regress on the covariate. Fig.2 illustrates this as a path diagram.

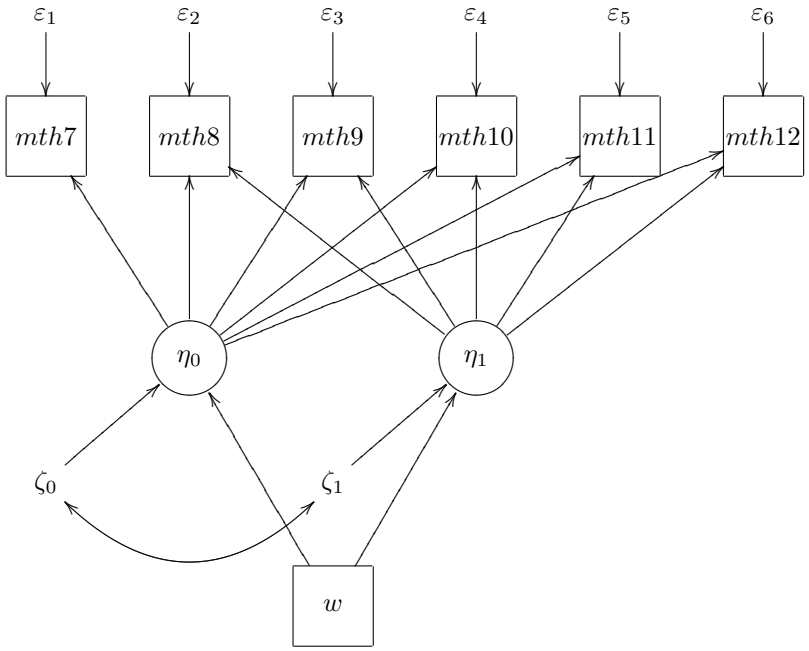

Figure 2: Path Diagram Illustrating Models 3 and 4

As summarized in Table 2, the χ^2 tests is statistically significant ($p < .001$) indicating a poor fit.

The CFI, however, is 0.990 which is above below the recommended threshold value for good model fit. Similarly, the TLI of 0.987 is also well above its threshold value for good fit. The model also clears the SRMSR threshold with 0.015.

The RMSEA of 0.076 is still slightly above its threshold for goodness of fit.

The composite result of these model fit statistics suggest that Model 3 fits the data well. The estimates and modification indices for Model 3 are summarized in Table 3 and Table 4 respectively.

With the introduction of the covariate into the model, the degrees of freedom available have increased. Some of the correlation between the "Intercept" and "Slope" factors is now being accounted for by the covariate. Table 9 indicate that the estimates of the two factors on the covariate `HOMERES` are statistically significant (Est./S.E. > 1.96). Furthermore, the covariance between the "intercept" and "slope" factors is also significant and positive, which indicates that the higher intercept a subject has, the steeper the slope.

It is important to note that the Modification Indices in Table 10 suggest that the "Slope" factor be regressed on the "Intercept" factor. If implemented, this would be a Latent Variable Regression.

Model 4

In this model, we simply replace the previous time-invariant covariate, `homeres`, with a different one: "Mother's Education" (`mothed`).

Since it is a simple substitution of a covariate, the equation set (4) and the path diagram Fig.2 both applies. The estimates and modification indices for Model 4 are summarized in Table 11 and Table 12 respectively.

From Table 2, it can be seen that the fit statistics for Model 4 is virtually identical to those of Model 3. We can therefore conclude that Model 4 fits the data just as well as Model 3.

Table 11 indicate that the estimates of the two factors on the covariate `MOTHED` are statistically significant (Est./S.E. > 1.96). Furthermore, the covariance between the "intercept" and "slope" factors is, once again, significant and positive, indicating that the higher intercept a subject has, the steeper the slope.

As it was the case with Model 3, the Modification Indices in Table 12 suggest that the "Slope" factor be regressed on the "Intercept" factor. If implemented, this would be a Latent Variable Regression.

Model 5

Finally, we use both covariates in the model to examine whether it would significantly improve the model fit comparing to Model 3 and 4, where only one of the covariates were present (paths for the other covariate were fixed to zero).

To reflect the inclusion of both time-invariant covariates in a mathematical notation, the equation set (4) will now be rewritten as follows:

$$
\begin{aligned}
y_{it} &= \boldsymbol{\nu} + \eta_{0i} \times \mathbf{1} + \eta_{1i} \times t + \varepsilon_{it} \qquad \text{where} \quad \nu = \alpha_0 \\
\\
\eta_{0i} &= \mathbf{0} + \gamma_{01} \times w_{1i} + \gamma_{02} \times w_{2i} + \zeta_{0i} \\
\eta_{1i} &= \alpha_1 + \gamma_{11} \times w_{1i} + \gamma_{12} \times w_{2i} + \zeta_{1i}
\end{aligned}
\tag{5}
$$

Table 9: Model 3 Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
MTHBETA0 |					
MTH7	1.000	0.000	0.000	9.402	0.919
MTH8	1.000	0.000	0.000	9.402	0.866
MTH9	1.000	0.000	0.000	9.402	0.778
MTH10	1.000	0.000	0.000	9.402	0.710
MTH11	1.000	0.000	0.000	9.402	0.671
MTH12	1.000	0.000	0.000	9.402	0.639
MTHBETA1 |					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	1.000	0.000	0.000	1.518	0.140
MTH9	2.367	0.108	21.905	3.594	0.297
MTH10	3.607	0.169	21.295	5.476	0.413
MTH11	4.363	0.208	20.975	6.622	0.473
MTH12	4.606	0.223	20.651	6.991	0.475
MTHBETA0 ON					
&HOMERES	2.470	0.199	12.388	0.263	0.431
MTHBETA1 ON					
&HOMERES	0.289	0.041	7.023	0.190	0.312
MTHBETA1 WITH					
MTHBETA0	4.125	0.601	6.859	0.289	0.289
Intercepts					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	0.000	0.000	0.000	0.000	0.000
MTH9	0.000	0.000	0.000	0.000	0.000
MTH10	0.000	0.000	0.000	0.000	0.000
MTH11	0.000	0.000	0.000	0.000	0.000
MTH12	0.000	0.000	0.000	0.000	0.000
MTHBETA0	45.411	0.740	61.384	4.830	4.830
MTHBETA1	2.685	0.199	13.482	1.769	1.769
Residual Variances					
MTH7	16.278	1.386	11.742	16.278	0.156
MTH8	15.050	1.041	14.454	15.050	0.128
MTH9	16.096	0.979	16.447	16.096	0.110
MTH10	13.576	0.917	14.811	13.576	0.077
MTH11	11.296	0.965	11.707	11.296	0.058
MTH12	23.257	1.527	15.231	23.257	0.108
MTHBETA0	71.967	4.190	17.177	0.814	0.814
MTHBETA1	2.080	0.271	7.682	0.903	0.903

Table 10: Model 3 Modification Indices

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.C.
BY Statements				
MTHBETA0 BY MTH7	14.580	-0.080	-0.749	-0.073
MTHBETA0 BY MTH11	6.684	-0.033	-0.312	-0.022
ON/BY Statements				
MTHBETA1 ON MTHBETA0 / MTHBETA0 BY MTHBETA1	999.000	0.000	0.000	0.000
WITH Statements				
MTH9 WITH MTH7	7.766	-2.488	-2.488	-0.020
MTH9 WITH MTH8	21.336	3.563	3.563	0.027
MTH10 WITH MTH9	31.802	4.077	4.077	0.025
MTH11 WITH MTH8	4.542	-1.529	-1.529	-0.010
MTH11 WITH MTH9	16.211	-3.048	-3.048	-0.018
MTH12 WITH MTH7	6.994	2.685	2.685	0.018
MTH12 WITH MTH9	10.337	-2.898	-2.898	-0.016
MTH12 WITH MTH10	17.146	-4.134	-4.134	-0.021
MTH12 WITH MTH11	44.338	7.968	7.968	0.039

Fig.3 illustrates this as a path diagram.

From Table 2, it can be seen that, with the exception of the χ^2 statistics and the increase in the degrees of freedom, the fit statistics for Model 5 is virtually identical to those of Model 3. We can therefore conclude that Model 5 fits the data well.

Table 11 indicate that the estimates of the two factors on the two covariates are statistically significant (Est./S.E. > 1.96). Furthermore, the covariance between the "intercept" and "slope" factors is also significant and positive, which indicates that the higher intercept a subject has, the steeper the slope.

Like with the proceeding Models 3 and 4, the Modification Indices in Table 14 suggest that the "Slope" factor be regressed on the "Intercept" factor as strongly.

Table 11: Model 4 Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
MTHBETA0 |					
MTH7	1.000	0.000	0.000	9.383	0.919
MTH8	1.000	0.000	0.000	9.383	0.866
MTH9	1.000	0.000	0.000	9.383	0.778
MTH10	1.000	0.000	0.000	9.383	0.710
MTH11	1.000	0.000	0.000	9.383	0.671
MTH12	1.000	0.000	0.000	9.383	0.638
MTHBETA1 |					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	1.000	0.000	0.000	1.535	0.142
MTH9	2.385	0.109	21.841	3.661	0.303
MTH10	3.598	0.169	21.232	5.522	0.418
MTH11	4.362	0.209	20.912	6.695	0.479
MTH12	4.610	0.224	20.580	7.074	0.481
MTHBETA0 ON					
MOTHED	3.385	0.340	9.970	0.361	0.357
MTHBETA1 ON					
MOTHED	0.253	0.067	3.796	0.165	0.164
MTHBETA1 WITH					
MTHBETA0	5.103	0.653	7.814	0.354	0.354
Intercepts					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	0.000	0.000	0.000	0.000	0.000
MTH9	0.000	0.000	0.000	0.000	0.000
MTH10	0.000	0.000	0.000	0.000	0.000
MTH11	0.000	0.000	0.000	0.000	0.000
MTH12	0.000	0.000	0.000	0.000	0.000
MTHBETA0	45.573	0.876	52.033	4.857	4.857
MTHBETA1	3.028	0.231	13.095	1.973	1.973
Residual Variances					
MTH7	16.252	1.389	11.703	16.252	0.156
MTH8	15.188	1.047	14.501	15.188	0.129
MTH9	15.802	0.962	16.425	15.802	0.109
MTH10	13.450	0.907	14.823	13.450	0.077
MTH11	10.977	0.951	11.539	10.977	0.056
MTH12	23.376	1.531	15.270	23.376	0.108
MTHBETA0	76.788	4.436	17.310	0.872	0.872
MTHBETA1	2.292	0.295	7.757	0.973	0.973

Table 12: Model 4 Modification Indices

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.
BY Statements				
MTHBETA0 BY MTH7	18.641	-0.086	-0.811	-0.079
MTHBETA0 BY MTH8	4.165	0.030	0.280	0.026
MTHBETA0 BY MTH10	4.467	0.022	0.206	0.016
MTHBETA0 BY MTH11	9.453	-0.038	-0.357	-0.026
ON/BY Statements				
MTHBETA1 ON MTHBETA0 / MTHBETA0 BY MTHBETA1	999.000	0.000	0.000	0.000
WITH Statements				
MTH9 WITH MTH7	7.556	-2.435	-2.435	-0.020
MTH9 WITH MTH8	15.895	3.056	3.056	0.023
MTH10 WITH MTH9	30.050	3.919	3.919	0.025
MTH11 WITH MTH8	5.085	-1.608	-1.608	-0.011
MTH11 WITH MTH9	15.761	-2.977	-2.977	-0.018
MTH12 WITH MTH7	7.616	2.806	2.806	0.019
MTH12 WITH MTH9	8.808	-2.659	-2.659	-0.015
MTH12 WITH MTH10	21.245	-4.591	-4.591	-0.024
MTH12 WITH MTH11	50.792	8.539	8.539	0.042

Table 13: Model 5 Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
MTHBETA0 |					
MTH7	1.000	0.000	0.000	9.369	0.917
MTH8	1.000	0.000	0.000	9.369	0.867
MTH9	1.000	0.000	0.000	9.369	0.778
MTH10	1.000	0.000	0.000	9.369	0.709
MTH11	1.000	0.000	0.000	9.369	0.671
MTH12	1.000	0.000	0.000	9.369	0.639
MTHBETA1 |					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	1.000	0.000	0.000	1.522	0.141
MTH9	2.385	0.111	21.449	3.630	0.301
MTH10	3.623	0.174	20.825	5.513	0.417
MTH11	4.385	0.214	20.507	6.673	0.478
MTH12	4.620	0.229	20.190	7.030	0.479
MTHBETA0 ON					
MOTHED	2.572	0.337	7.622	0.275	0.270
HOMERES	2.004	0.204	9.805	0.214	0.348
MTHBETA1 ON					
MOTHED	0.161	0.067	2.389	0.106	0.104
HOMERES	0.265	0.043	6.178	0.174	0.283
MTHBETA1 WITH					
MTHBETA0	3.650	0.581	6.278	0.256	0.256
Intercepts					
MTH7	0.000	0.000	0.000	0.000	0.000
MTH8	0.000	0.000	0.000	0.000	0.000
MTH9	0.000	0.000	0.000	0.000	0.000
MTH10	0.000	0.000	0.000	0.000	0.000
MTH11	0.000	0.000	0.000	0.000	0.000
MTH12	0.000	0.000	0.000	0.000	0.000
MTHBETA0	40.879	0.956	42.767	4.363	4.363
MTHBETA1	2.347	0.227	10.339	1.542	1.542
Residual Variances					
MTH7	16.500	1.417	11.646	16.500	0.158
MTH8	15.021	1.056	14.221	15.021	0.128
MTH9	15.929	0.983	16.206	15.929	0.110
MTH10	13.504	0.924	14.610	13.504	0.077
MTH11	11.094	0.970	11.436	11.094	0.057
MTH12	23.324	1.549	15.060	23.324	0.108
MTHBETA0	66.259	3.955	16.751	0.755	0.755
MTHBETA1	2.067	0.275	7.532	0.893	0.893

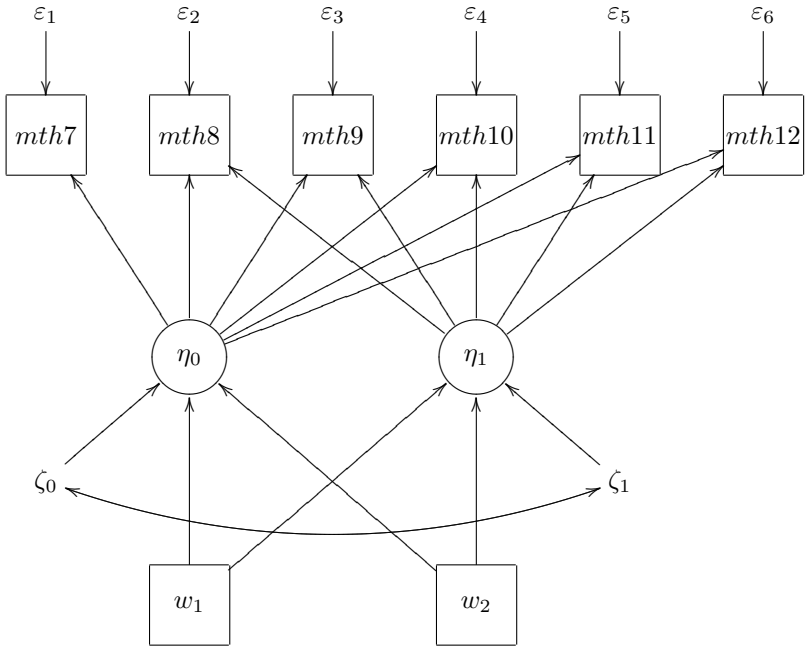

Figure 3: Path Diagram Illustrating Model 5

Table 14: Model 5 Modification Indices

	M.I.	E.P.C.	Std E.P.C.	StdYX E.P.
BY Statements				
MTHBETA0 BY MTH7	16.906	-0.084	-0.788	-0.077
MTHBETA0 BY MTH11	8.472	-0.037	-0.346	-0.025
ON/BY Statements				
MTHBETA1 ON MTHBETA0 / MTHBETA0 BY MTHBETA1	999.000	0.000	0.000	0.000
WITH Statements				
MTH9 WITH MTH7	7.285	-2.439	-2.439	-0.020
MTH9 WITH MTH8	17.416	3.249	3.249	0.025
MTH10 WITH MTH9	30.918	4.047	4.047	0.025
MTH11 WITH MTH8	4.761	-1.578	-1.578	-0.010
MTH11 WITH MTH9	16.241	-3.072	-3.072	-0.018
MTH12 WITH MTH7	6.942	2.723	2.723	0.018
MTH12 WITH MTH9	10.020	-2.880	-2.880	-0.016
MTH12 WITH MTH10	19.276	-4.430	-4.430	-0.023
MTH12 WITH MTH11	44.608	8.087	8.087	0.039

M*plus* Code: Linear Growth, No Covariate

```
Title:  This is a partial input file for HW1 that will now be used
        for HW3.  The variable names are for the data set lsay.dat.

        Repeated Mesures Outcome:   Aggregate Math IRT Score 1.
        Growth Curve:               Linear.
        Covariates:                 None.

DATA:   FILE IS C:\Ed.231E\Assignment_03\lsay.dat;

VARIABLE:  Names are lsayid schcode classize urban tracking ntracks
             mthlvl female mthflg7-mthflg12 mothed fathed mothsei
             fathsei homeres race
             expect parapsh parcpsh parmpsh peerapsh peermpsh
             bas7 basse7 alg7 algse7 geo7 geose7
             qlt7 qltse7 mth7 mthse7 mtha7 mthase7
             bas8 basse8 alg8 algse8 geo8 geose8
             qlt8 qltse8 mth8 mthse8 mtha8 mthase8
             bas9 basse9 alg9 algse9 geo9 geose9
             qlt9 qltse9 mth9 mthse9 mtha9 mthase9
             bas10 basse10 alg10 algse10 geo10 geose10
             qlt10 qltse10 mth10 mthse10 mtha10 mthase10
             bas11 basse11 alg11 algse11 geo11 geose11
             qlt11 qltse11 mth11 mthse11 mtha11 mthase11
             bas12 basse12 alg12 algse12 geo12 geose12
             qlt12 qltse12 mth12 mthse12 mtha12 mthase12
             mthcrs7-mthcrs12 mtrk10-mtrk12 totstud lchfull
             lchpart parvis mcirr mclub strat mstrat comp mcomp
             african hispan asian expel arrest dropot self worth
             other satisf respect failure esteem problem cloctn
             dloctn eloctn floctn gloctn hloctn iloctn jloctn
             kloctn lloctn drink runawa suicid alc7 alc10 alc11
             alc12 arest7 runa8 runa9 runa10 runa11 run12 suic8
             suic9 suic10 suic11 suic12 drop7 drop8 drop9 drop10
             drop11 drop12 fdrop8 fdrop9 fdrop10 fdrop11 fdrop12
             enj7 good7 und7 useboy7 nerv7 wor7 scar7 use7 logic7
             boybet7 job7 often7 enj8 good8 und8 useboy8 nerv8
             wor8 scar8 use8 logic8 boybet8 job8 often8 enj9
             good9 und9 useboy9 nerv9 wor9 scar9 use9 logic9
             boybet9 job9 often9 enj10 good10 und10 useboy10
             nerv10 wor10 scar10 use10 logic10 boybet10 job10
             often10;
           !
           ! End of Variable List
```

```
            !

        USEVAR= !bas7 bas8 bas9 bas10 bas11 bas12;
                ! -- Basic math IRT scores.
                !
                !alg7 alg8 alg9 alg10 alg11 alg12;
                ! -- Algebra IRT score.
                !
                !geo7 geo8 geo9 geo10 geo11 geo12;
                ! -- Geometry IRT score
                !
                !qlt7 qlt8 qlt9 qlt10 qlt11 qlt12;
                ! -- Quantitative Literacy IRT
                !
                mth7 mth8 mth9 mth10 mth11 mth12;
                ! -- Aggregate math IRT, w/o aberrants
                !
                !mtha7 mtha8 mtha9 mtha10 mtha11 matha12;
                ! -- Aggregate math IRT, w/ aberrants

        Missing are all(9999);

ANALYSIS:   TYPE = MEANSTRUCTURE;
            ESTIMATOR = ML;

MODEL:  ! Mplus Version 3 Language:
mthbeta0 mthbeta1 | mth7@0 mth8@1 mth9@2 mth10@3 mth11@4 mth12@5;


PLOT: TYPE = plot1 plot2 plot3;
        ! plot1 ... Summary Statistics.
        ! plot2 ... Estimatimation Results.

OUTPUT: SAMPSTAT STANDARDIZED MODINDICES(3.84) FSDeterminacy;
        ! SAMPSTAT ... produces sample statistics for the data
        !              that is being analyzed.
        !
        ! Standardized ... produces standardized coefficients .
        !
        ! Modindices ... produced modification indeces with
        !                the minimum expected drop in the
        !                chi-squre given in the paranthesis.
        !
        !                While some use (0), others put donw
        !                (3.84) which would exclue those drops
        !                that are non-significant.
        !
        ! FSDeterminacy...produces Factor Score Determinacies.
        !                 FSDeterminacy indicates how close
        !                 the average estimate factor scores
        !                 are to true factor scores.
        !
        !                 In general, it is desirable to have
        !                 values that are .80 or higher.
        !
```

M*plus* Code: Non- Linear Growth, No Covariate

```
Title:  This is a partial input file for HW1 that will now be used
        for HW3.  The variable names are for the data set lsay.dat.

        Repeated Mesures Outcome:   Aggregate Math IRT Score 1.
        Growth Curve:               Non Linear.
        Covariates:                 None.

DATA:   FILE IS C:\Ed.231E\Assignment_03\lsay.dat;

VARIABLE:  Names are lsayid schcode classize urban tracking ntracks
             mthlvl female mthflg7-mthflg12 mothed fathed mothsei
             fathsei homeres race
             expect parapsh parcpsh parmpsh peerapsh peermpsh
             bas7 basse7 alg7 algse7 geo7 geose7
             qlt7 qltse7 mth7 mthse7 mtha7 mthase7
             bas8 basse8 alg8 algse8 geo8 geose8
             qlt8 qltse8 mth8 mthse8 mtha8 mthase8
             bas9 basse9 alg9 algse9 geo9 geose9
             qlt9 qltse9 mth9 mthse9 mtha9 mthase9
             bas10 basse10 alg10 algse10 geo10 geose10
             qlt10 qltse10 mth10 mthse10 mtha10 mthase10
             bas11 basse11 alg11 algse11 geo11 geose11
             qlt11 qltse11 mth11 mthse11 mtha11 mthase11
             bas12 basse12 alg12 algse12 geo12 geose12
             qlt12 qltse12 mth12 mthse12 mtha12 mthase12
             mthcrs7-mthcrs12 mtrk10-mtrk12 totstud lchfull
             lchpart parvis mcirr mclub strat mstrat comp mcomp
             african hispan asian expel arrest dropot self worth
             other satisf respect failure esteem problem cloctn
             dloctn eloctn floctn gloctn hloctn iloctn jloctn
             kloctn lloctn drink runawa suicid alc7 alc10 alc11
             alc12 arest7 runa8 runa9 runa10 runa11 run12 suic8
             suic9 suic10 suic11 suic12 drop7 drop8 drop9 drop10
             drop11 drop12 fdrop8 fdrop9 fdrop10 fdrop11 fdrop12
             enj7 good7 und7 useboy7 nerv7 wor7 scar7 use7 logic7
             boybet7 job7 often7 enj8 good8 und8 useboy8 nerv8
             wor8 scar8 use8 logic8 boybet8 job8 often8 enj9
             good9 und9 useboy9 nerv9 wor9 scar9 use9 logic9
             boybet9 job9 often9 enj10 good10 und10 useboy10
             nerv10 wor10 scar10 use10 logic10 boybet10 job10
             often10;
           !
           ! End of Variable List
```

```
            !

        USEVAR= !bas7 bas8 bas9 bas10 bas11 bas12;
                ! -- Basic math IRT scores.
                !
                !alg7 alg8 alg9 alg10 alg11 alg12;
                ! -- Algebra IRT score.
                !
                !geo7 geo8 geo9 geo10 geo11 geo12;
                ! -- Geometry IRT score
                !
                !qlt7 qlt8 qlt9 qlt10 qlt11 qlt12;
                ! -- Quantitative Literacy IRT
                !
                mth7 mth8 mth9 mth10 mth11 mth12;
                ! -- Aggregate math IRT, w/o aberrants
                !
                !mtha7 mtha8 mtha9 mtha10 mtha11 matha12;
                ! -- Aggregate math IRT, w/ aberrants

        Missing are all(9999);

ANALYSIS:   TYPE = MEANSTRUCTURE;
            ESTIMATOR = ML;

MODEL:  ! Mplus Version 3 Language:
mthbeta0 mthbeta1 | mth7@0 mth8@1 mth9 mth10 mth11 mth12;
! Allows for non-linear growth curve.

PLOT: TYPE = plot1 plot2 plot3;
        ! plot1 ... Summary Statistics.
        ! plot2 ... Estimatimation Results.

OUTPUT: SAMPSTAT STANDARDIZED MODINDICES(3.84) FSDeterminacy;
        ! SAMPSTAT ... produces sample statistics for the data
        !              that is being analyzed.
        !
        ! Standardized ... produces standardized coefficients .
        !
        ! Modindices ... produced modification indeces with
        !                the minimum expected drop in the
        !                chi-squre given in the paranthesis.
        !
        !                While some use (0), others put donw
        !                (3.84) which would exclue those drops
        !                that are non-significant.
        !
        ! FSDeterminacy...produces Factor Score Determinacies.
        !                 FSDeterminacy indicates how close
        !                 the average estimate factor scores
        !                 are to true factor scores.
        !
        !                 In general, it is desirable to have
        !                 values that are .80 or higher.
        !
```

M*plus* Code: Non- Linear Growth, Covariate ... "homeres"

```
Title:  This is a partial input file for HW1 that will now be used
        for HW3.  The variable names are for the data set lsay.dat.

        Repeated Mesures Outcome:   Aggregate Math IRT Score 1.
        Growth Curve:               Non Linear.
        Covariates:                 homeres.

DATA:   FILE IS C:\Ed.231E\Assignment_03\lsay.dat;

VARIABLE:  Names are lsayid schcode classize urban tracking ntracks
             mthlvl female mthflg7-mthflg12 mothed fathed mothsei
             fathsei homeres race
             expect parapsh parcpsh parmpsh peerapsh peermpsh
             bas7 basse7 alg7 algse7 geo7 geose7
             qlt7 qltse7 mth7 mthse7 mtha7 mthase7
             bas8 basse8 alg8 algse8 geo8 geose8
             qlt8 qltse8 mth8 mthse8 mtha8 mthase8
             bas9 basse9 alg9 algse9 geo9 geose9
             qlt9 qltse9 mth9 mthse9 mtha9 mthase9
             bas10 basse10 alg10 algse10 geo10 geose10
             qlt10 qltse10 mth10 mthse10 mtha10 mthase10
             bas11 basse11 alg11 algse11 geo11 geose11
             qlt11 qltse11 mth11 mthse11 mtha11 mthase11
             bas12 basse12 alg12 algse12 geo12 geose12
             qlt12 qltse12 mth12 mthse12 mtha12 mthase12
             mthcrs7-mthcrs12 mtrk10-mtrk12 totstud lchfull
             lchpart parvis mcirr mclub strat mstrat comp mcomp
             african hispan asian expel arrest dropot self worth
             other satisf respect failure esteem problem cloctn
             dloctn eloctn floctn gloctn hloctn iloctn jloctn
             kloctn lloctn drink runawa suicid alc7 alc10 alc11
             alc12 arest7 runa8 runa9 runa10 runa11 run12 suic8
             suic9 suic10 suic11 suic12 drop7 drop8 drop9 drop10
             drop11 drop12 fdrop8 fdrop9 fdrop10 fdrop11 fdrop12
             enj7 good7 und7 useboy7 nerv7 wor7 scar7 use7 logic7
             boybet7 job7 often7 enj8 good8 und8 useboy8 nerv8
             wor8 scar8 use8 logic8 boybet8 job8 often8 enj9
             good9 und9 useboy9 nerv9 wor9 scar9 use9 logic9
             boybet9 job9 often9 enj10 good10 und10 useboy10
             nerv10 wor10 scar10 use10 logic10 boybet10 job10
             often10;
           !
           ! End of Variable List
```

```
            !

        USEVAR= !bas7 bas8 bas9 bas10 bas11 bas12;
                ! -- Basic math IRT scores.
                !
                !alg7 alg8 alg9 alg10 alg11 alg12;
                ! -- Algebra IRT score.
                !
                !geo7 geo8 geo9 geo10 geo11 geo12;
                ! -- Geometry IRT score
                !
                !qlt7 qlt8 qlt9 qlt10 qlt11 qlt12;
                ! -- Quantitative Literacy IRT
                !
                homeres mth7 mth8 mth9 mth10 mth11 mth12;
                ! -- Aggregate math IRT, w/o aberrants
                !
                !mtha7 mtha8 mtha9 mtha10 mtha11 matha12;
                ! -- Aggregate math IRT, w/ aberrants

        Missing are all(9999);

ANALYSIS:   TYPE = MEANSTRUCTURE;
            ESTIMATOR = ML;

MODEL:  ! Mplus Version 3 Language:
mthbeta0 mthbeta1 | mth7@0 mth8@1 mth9 mth10 mth11 mth12;
mthbeta0 mthbeta1 ON homeres;
! Allows for non-linear growth curve.
!
! Slope and Intercept parameters (factors) are now regressed on
! a newly introduced covariate: homeres.


PLOT: TYPE = plot1 plot2 plot3;
        ! plot1 ... Summary Statistics.
        ! plot2 ... Estimatimation Results.

OUTPUT: SAMPSTAT STANDARDIZED MODINDICES(3.84) FSDeterminacy;
        ! SAMPSTAT ... produces sample statistics for the data
        !              that is being analyzed.
        !
        ! Standardized ... produces standardized coefficients .
        !
        ! Modindices ... produced modification indeces with
        !                the minimum expected drop in the
        !                chi-squre given in the paranthesis.
        !
        !                While some use (0), others put donw
        !                (3.84) which would exclue those drops
        !                that are non-significant.
        !
        ! FSDeterminacy...produces Factor Score Determinacies.
        !                 FSDeterminacy indicates how close
        !                 the average estimate factor scores
```

```
!                are to true factor scores.
!
!                In general, it is desirable to have
!                values that are .80 or higher.
!
```

M*plus* Code: Non- Linear Growth, Covariate ... "mothed"

```
Title:  This is a partial input file for HW1 that will now be used
        for HW3.  The variable names are for the data set lsay.dat.

        Repeated Mesures Outcome:   Aggregate Math IRT Score 1.
        Growth Curve:               Non Linear.
        Covariates:                 mothed.

DATA:   FILE IS C:\Ed.231E\Assignment_03\lsay.dat;

VARIABLE:  Names are lsayid schcode classize urban tracking ntracks
             mthlvl female mthflg7-mthflg12 mothed fathed mothsei
             fathsei homeres race
             expect parapsh parcpsh parmpsh peerapsh peermpsh
             bas7 basse7 alg7 algse7 geo7 geose7
             qlt7 qltse7 mth7 mthse7 mtha7 mthase7
             bas8 basse8 alg8 algse8 geo8 geose8
             qlt8 qltse8 mth8 mthse8 mtha8 mthase8
             bas9 basse9 alg9 algse9 geo9 geose9
             qlt9 qltse9 mth9 mthse9 mtha9 mthase9
             bas10 basse10 alg10 algse10 geo10 geose10
             qlt10 qltse10 mth10 mthse10 mtha10 mthase10
             bas11 basse11 alg11 algse11 geo11 geose11
             qlt11 qltse11 mth11 mthse11 mtha11 mthase11
             bas12 basse12 alg12 algse12 geo12 geose12
             qlt12 qltse12 mth12 mthse12 mtha12 mthase12
             mthcrs7-mthcrs12 mtrk10-mtrk12 totstud lchfull
             lchpart parvis mcirr mclub strat mstrat comp mcomp
             african hispan asian expel arrest dropot self worth
             other satisf respect failure esteem problem cloctn
             dloctn eloctn floctn gloctn hloctn iloctn jloctn
             kloctn lloctn drink runawa suicid alc7 alc10 alc11
             alc12 arest7 runa8 runa9 runa10 runa11 run12 suic8
             suic9 suic10 suic11 suic12 drop7 drop8 drop9 drop10
             drop11 drop12 fdrop8 fdrop9 fdrop10 fdrop11 fdrop12
             enj7 good7 und7 useboy7 nerv7 wor7 scar7 use7 logic7
             boybet7 job7 often7 enj8 good8 und8 useboy8 nerv8
             wor8 scar8 use8 logic8 boybet8 job8 often8 enj9
             good9 und9 useboy9 nerv9 wor9 scar9 use9 logic9
             boybet9 job9 often9 enj10 good10 und10 useboy10
             nerv10 wor10 scar10 use10 logic10 boybet10 job10
             often10;
           !
           ! End of Variable List
```

```
            !

        USEVAR= !bas7 bas8 bas9 bas10 bas11 bas12;
                ! -- Basic math IRT scores.
                !
                !alg7 alg8 alg9 alg10 alg11 alg12;
                ! -- Algebra IRT score.
                !
                !geo7 geo8 geo9 geo10 geo11 geo12;
                ! -- Geometry IRT score
                !
                !qlt7 qlt8 qlt9 qlt10 qlt11 qlt12;
                ! -- Quantitative Literacy IRT
                !
                mothed mth7 mth8 mth9 mth10 mth11 mth12;
                ! -- Aggregate math IRT, w/o aberrants
                !
                !mtha7 mtha8 mtha9 mtha10 mtha11 matha12;
                ! -- Aggregate math IRT, w/ aberrants

        Missing are all(9999);

ANALYSIS:   TYPE = MEANSTRUCTURE;
            ESTIMATOR = ML;

MODEL:  ! Mplus Version 3 Language:
mthbeta0 mthbeta1 | mth7@0 mth8@1 mth9 mth10 mth11 mth12;
mthbeta0 mthbeta1 ON mothed;
! Allows for non-linear growth curve.
!
! Slope and Intercept parameters (factors) are now regressed on
! a newly introduced covariate: mothed.


PLOT: TYPE = plot1 plot2 plot3;
        ! plot1 ... Summary Statistics.
        ! plot2 ... Estimatimation Results.

OUTPUT: SAMPSTAT STANDARDIZED MODINDICES(3.84) FSDeterminacy;
        ! SAMPSTAT ... produces sample statistics for the data
        !              that is being analyzed.
        !
        ! Standardized ... produces standardized coefficients .
        !
        ! Modindices ... produced modification indeces with
        !                the minimum expected drop in the
        !                chi-squre given in the paranthesis.
        !
        !                While some use (0), others put donw
        !                (3.84) which would exclue those drops
        !                that are non-significant.
        !
        ! FSDeterminacy...produces Factor Score Determinacies.
        !                 FSDeterminacy indicates how close
        !                 the average estimate factor scores
```

```
!                are to true factor scores.
!
!                In general, it is desirable to have
!                values that are .80 or higher.
!
```

M*plus* Code: Non- Linear Growth, Covariate ... "homeres" and "mothed"

```
Title:  This is a partial input file for HW1 that will now be used
        for HW3.  The variable names are for the data set lsay.dat.

        Repeated Mesures Outcome:   Aggregate Math IRT Score 1.
        Growth Curve:               Non Linear.
        Covariates:                 mothed homeres.

DATA:   FILE IS C:\Ed.231E\Assignment_03\lsay.dat;

VARIABLE:  Names are lsayid schcode classize urban tracking ntracks
             mthlvl female mthflg7-mthflg12 mothed fathed mothsei
             fathsei homeres race
             expect parapsh parcpsh parmpsh peerapsh peermpsh
             bas7 basse7 alg7 algse7 geo7 geose7
             qlt7 qltse7 mth7 mthse7 mtha7 mthase7
             bas8 basse8 alg8 algse8 geo8 geose8
             qlt8 qltse8 mth8 mthse8 mtha8 mthase8
             bas9 basse9 alg9 algse9 geo9 geose9
             qlt9 qltse9 mth9 mthse9 mtha9 mthase9
             bas10 basse10 alg10 algse10 geo10 geose10
             qlt10 qltse10 mth10 mthse10 mtha10 mthase10
             bas11 basse11 alg11 algse11 geo11 geose11
             qlt11 qltse11 mth11 mthse11 mtha11 mthase11
             bas12 basse12 alg12 algse12 geo12 geose12
             qlt12 qltse12 mth12 mthse12 mtha12 mthase12
             mthcrs7-mthcrs12 mtrk10-mtrk12 totstud lchfull
             lchpart parvis mcirr mclub strat mstrat comp mcomp
             african hispan asian expel arrest dropot self worth
             other satisf respect failure esteem problem cloctn
             dloctn eloctn floctn gloctn hloctn iloctn jloctn
             kloctn lloctn drink runawa suicid alc7 alc10 alc11
             alc12 arest7 runa8 runa9 runa10 runa11 run12 suic8
             suic9 suic10 suic11 suic12 drop7 drop8 drop9 drop10
             drop11 drop12 fdrop8 fdrop9 fdrop10 fdrop11 fdrop12
             enj7 good7 und7 useboy7 nerv7 wor7 scar7 use7 logic7
             boybet7 job7 often7 enj8 good8 und8 useboy8 nerv8
             wor8 scar8 use8 logic8 boybet8 job8 often8 enj9
             good9 und9 useboy9 nerv9 wor9 scar9 use9 logic9
             boybet9 job9 often9 enj10 good10 und10 useboy10
             nerv10 wor10 scar10 use10 logic10 boybet10 job10
             often10;
           !
           ! End of Variable List
```

```
            !

        USEVAR= !bas7 bas8 bas9 bas10 bas11 bas12;
                ! -- Basic math IRT scores.
                !
                !alg7 alg8 alg9 alg10 alg11 alg12;
                ! -- Algebra IRT score.
                !
                !geo7 geo8 geo9 geo10 geo11 geo12;
                ! -- Geometry IRT score
                !
                !qlt7 qlt8 qlt9 qlt10 qlt11 qlt12;
                ! -- Quantitative Literacy IRT
                !
                mothed homeres mth7 mth8 mth9 mth10 mth11 mth12;
                ! -- Aggregate math IRT, w/o aberrants
                !
                !mtha7 mtha8 mtha9 mtha10 mtha11 matha12;
                ! -- Aggregate math IRT, w/ aberrants

        Missing are all(9999);

ANALYSIS:   TYPE = MEANSTRUCTURE;
            ESTIMATOR = ML;

MODEL:  ! Mplus Version 3 Language:
mthbeta0 mthbeta1 | mth7@0 mth8@1 mth9 mth10 mth11 mth12;
mthbeta0 mthbeta1 ON mothed homeres;
! Allows for non-linear growth curve.
!
! Slope and Intercept parameters (factors) are now regressed on
! newly introduced covariates: mothed homeres.


PLOT: TYPE = plot1 plot2 plot3;
        ! plot1 ... Summary Statistics.
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OUTPUT: SAMPSTAT STANDARDIZED MODINDICES(3.84) FSDeterminacy;
        ! SAMPSTAT ... produces sample statistics for the data
        !              that is being analyzed.
        !
        ! Standardized ... produces standardized coefficients .
        !
        ! Modindices ... produced modification indeces with
        !                the minimum expected drop in the
        !                chi-squre given in the paranthesis.
        !
        !                While some use (0), others put donw
        !                (3.84) which would exclue those drops
        !                that are non-significant.
        !
        ! FSDeterminacy...produces Factor Score Determinacies.
        !                 FSDeterminacy indicates how close
        !                 the average estimate factor scores
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!                are to true factor scores.
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!                In general, it is desirable to have
!                values that are .80 or higher.
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