Typical Examples Of Growth Modeling

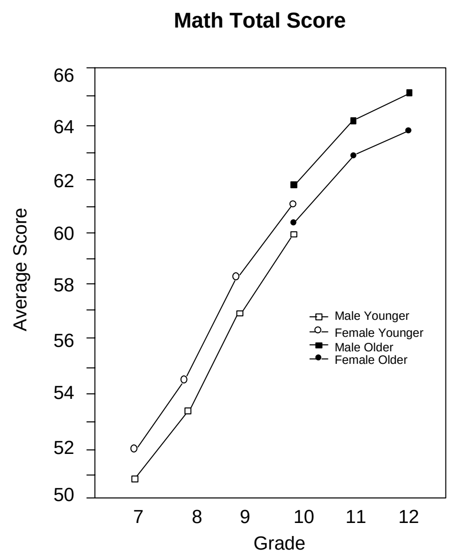
1

LSAY Data

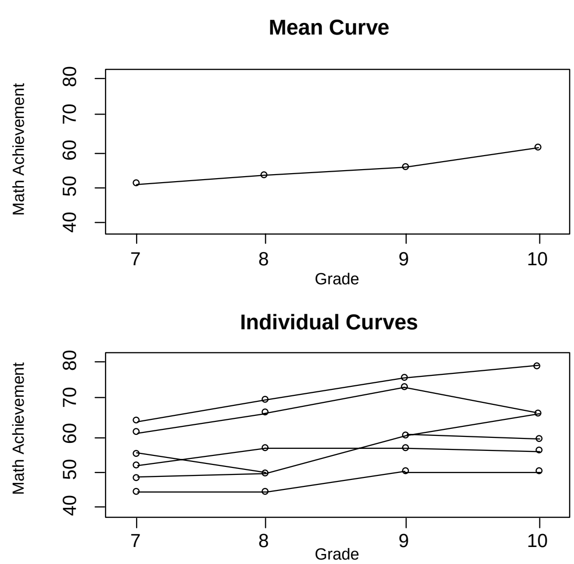
Longitudinal Study of American Youth (LSAY)

- Two cohorts measured each year beginning in 1987
  - Cohort 1 - Grades 10, 11, and 12
  - Cohort 2 - Grades 7, 8, 9, and 10
- Each cohort contains approximately 60 schools with approximately 60 students per school
- Variables - math and science achievement items, math and science attitude measures, and background variables from parents, teachers, and school principals
- Approximately 60 items per test with partial item overlap across grades - adaptive tests

2

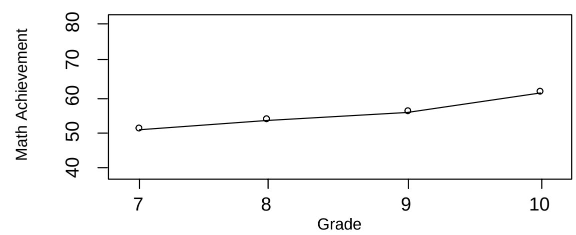

Math Total Score
Average Score
66
64
62
60
58
56
54
52
50
7
8
9
10
11
12
Grade
Male Younger
Female Younger
Male Older
Female Older

3

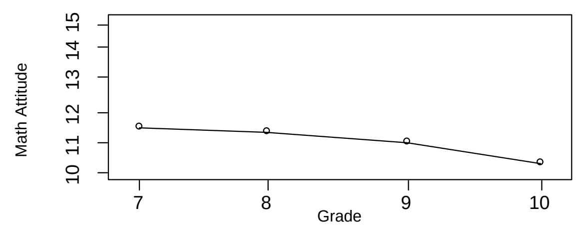

Mean Curve
Math Achievement
80
70
60
50
40
7
8
9
10
Grade
Individual Curves
Math Achievement
80
70
60
50
40
7
8
9
10
Grade

4

Sample Means for Math



Sample Means for Attitude Towards Math



5

Maternal Health Project Data

Maternal Health Project (MHP)

- Mothers who drank at least three drinks a week during their first trimester plus a random sample of mothers who used alcohol less often
- Mothers measured at fourth month and seventh month of pregnancy, at delivery, and at 8, 18, and 36 months postpartum
- Offspring measured at 0, 8, 18 and 36 months
- Variables for mothers - demographic, lifestyle, current environment, medical history, maternal psychological status, alcohol use, tobacco use, marijuana use, other illicit drug use
- Variables for offspring - head circumference, height, weight, gestational age, gender, and ethnicity

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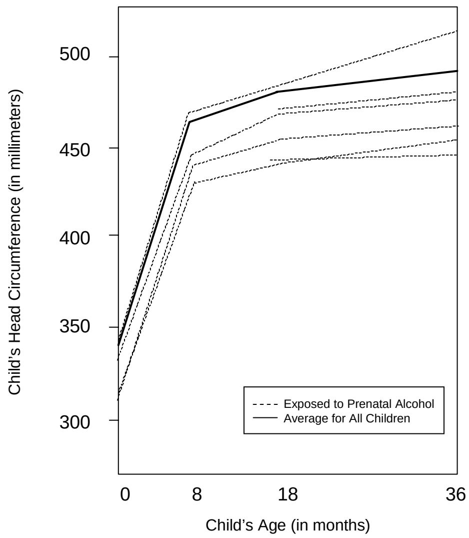
MHP: Offspring Head Circumference



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Basic Modeling Ideas

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Longitudinal Data: Three Approaches

Three modeling approaches for the regression of outcome on time (n is sample size, T is number of timepoints):

- **Use all *n* x *T* data points to do a single regression analysis**: Gives an intercept and a slope estimate for all individuals - does not account for individual differences or lack of independence of observations
- **Use each individual's *T* data points to do *n* regression analyses:** Gives an intercept and a slope estimate for each individual. Accounts for individual differences, but does not account for similarities among individuals
- **Use all *n* x *T* data points to do a single random effect regression analysis:** Gives an intercept and a slope estimate for each individual. Accounts for similarities among individuals by stipulating that all individuals' random effects come from a single, common population and models the non-independence of observations as show on the next page

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Individual Development Over Time

(1) $y_{ti} = \eta_{0i} + \eta_{1i} \, x_t + \varepsilon_{ti}$

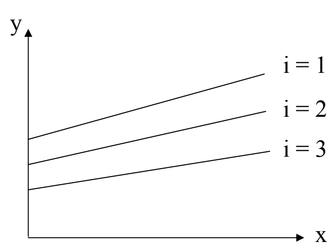
(2a) $\eta_{0i} = \alpha_0 + \gamma_0 \, w_i + \zeta_{0i}$

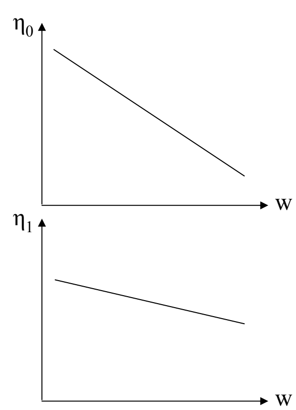
(2b) $\eta_{1i} = \alpha_1 + \gamma_1 \, w_i + \zeta_{1i}$

t = timepoint i = individual w = time-invariant covariate

y = outcome x = time score

η_0 = intercept η_1 = slope





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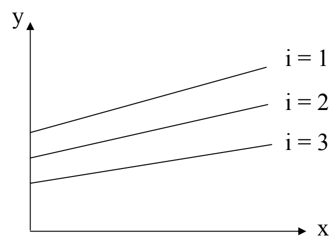
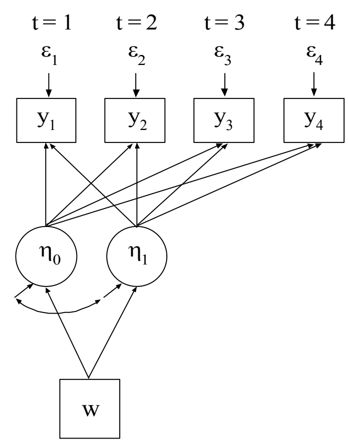
Individual Development Over Time





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(2a) $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$

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Growth Modeling Frameworks

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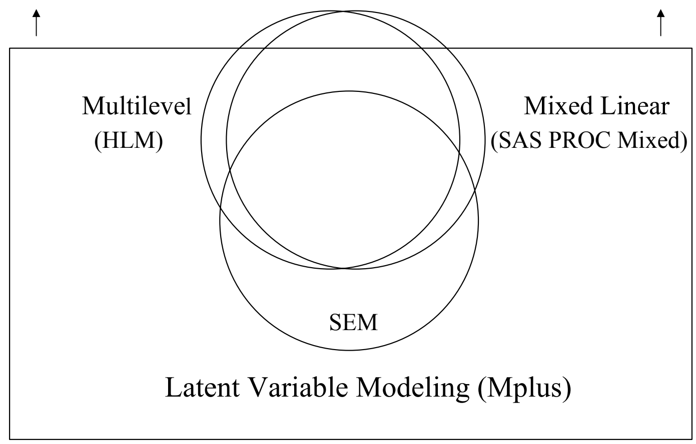
Growth Modeling Frameworks/Software



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Comparison Summary Of Multilevel, Mixed Linear, And SEM Growth Models

- Multilevel and mixed linear models are the same
- SEM differs from the multilevel and mixed linear models in two ways
  - Treatment of time scores
    - Time scores are data for multilevel and mixed linear models -- individuals can have different times of measurement
    - Time scores are parameters for SEM growth models -- time scores can be estimated
  - Treatment of time-varying covariates
    - Time-varying covariates have random effect coefficients for multilevel and mixed linear models -- coefficients vary over individuals
    - Time-varying covariates have fixed effect coefficients for SEM growth models -- coefficients vary over time

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Random Effects: Multilevel And Mixed Linear Modeling

Individual i ($i = 1, 2, \ldots, n$) observed at time point t ($t = 1, 2, \ldots T$).

Multilevel model with two levels (e.g. Raudenbush & Bryk, 2002, HLM).

- Level 1: $y_{ti} = \eta_{0i} + \eta_{1i}\, x_{ti} + \kappa_i\, w_{ti} + \varepsilon_{ti}$ (39)
- Level 2: $\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}$ (40)
  $\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}$ (41)
  $\kappa_i = \alpha + \gamma\, w_i + \zeta_i$ (42)

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Random Effects: Multilevel And Mixed Linear Modeling (Continued)

Mixed linear model:

$$y_{ti} = \textit{fixed part} + \textit{random part} \quad (43)$$
$$= \alpha_0 + \gamma_0\, w_i + (\alpha_1 + \gamma_1\, w_i)\, x_{ti} + (\alpha + \gamma\, w_i)\, w_{ti} \quad (44)$$
$$+ \zeta_{0i} + \zeta_{1i}\, x_{ti} + \zeta_i\, w_{ti} + \varepsilon_{ti}\,. \quad (45)$$

E.g. "*time* x w_i" refers to γ_1 (e.g. Rao, 1958; Laird & Ware, 1982; Jennrich & Sluchter, 1986; Lindstrom & Bates, 1988; BMDP5V; Goldstein, 1995, MLn; SAS PROC MIXED - Littell et al. 1996 and Singer, 1999).

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Random Effects: SEM And Multilevel Modeling

SEM (Tucker, 1958; Meredith & Tisak, 1990; McArdle & Epstein 1987; SEM software):

Measurement part:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \kappa_t\, w_{ti} + \varepsilon_{ti}\,. \tag{46}$$

Compare with level 1 of multilevel:

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_{ti} + \kappa_i\, w_{ti} + \varepsilon_{ti}\,. \tag{47}$$

Multilevel approach:

- x_{ti} as data: Flexible individually-varying times of observation
- Slopes for time-varying covariates vary over individuals

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Random Effects: SEM And Multilevel Modeling (Continued)

SEM approach:

- x_t as parameters: Flexible growth function form
- Slopes for time-varying covariates vary over time points

Structural part (same as level 2, except for κ_t):

$$\eta_{0i} = \alpha_0 + \gamma_0\, w_i + \zeta_{0i}\,, \tag{48}$$

$$\eta_{1i} = \alpha_1 + \gamma_1\, w_i + \zeta_{1i}\,, \tag{49}$$

κ_t not involved (parameter).

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Random Effects: Mixed Linear Modeling And SEM

Mixed linear model in matrix form:

$$\boldsymbol{y}_i = (y_{1i}, y_{2i}, \ldots, y_{Ti})' \qquad (51)$$

$$= \boldsymbol{X}_i \, \boldsymbol{\alpha} + \boldsymbol{Z}_i \, \boldsymbol{b}_i + \boldsymbol{e}_i \, . \qquad (52)$$

Here, ***X, Z*** are design matrices with known values, $\boldsymbol{\alpha}$ contains fixed effects, and ***b*** contains random effects. Compare with (43) - (45).

Random Effects: Mixed Linear Modeling And SEM (Continued)

SEM in matrix form:

$$y_i = \nu + \Lambda \, \eta_i + K \, x_i + \varepsilon_i \, , \qquad (53)$$

$$\eta_i = \alpha + B \, \eta_i + \Gamma \, x_i + \zeta_i \, . \qquad (54)$$

$$y_i = \textit{fixed part} + \textit{random part}$$

$$= \nu + \Lambda \, (I - B)^{-1} \, \alpha + \Lambda \, (I - B)^{-1} \, \Gamma \, x_i + K \, x_i$$

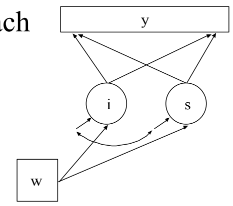
$$+ \Lambda \, (I - B)^{-1} \, \zeta_i + \varepsilon_i \, .$$

Assume $x_{ti} = x_t$, $\kappa_i = \kappa_t$ in (39). Then (39) is handled by (53) and (40) – (41) are handled by (54), putting x_t in Λ and w_{ti}, w_i in x_i.

Need for Λ_i, K_i, B_i, Γ_i.

Growth Modeling Approached In Two Ways: Data Arranged As Wide Versus Long

- Wide: Multivariate, Single-Level Approach

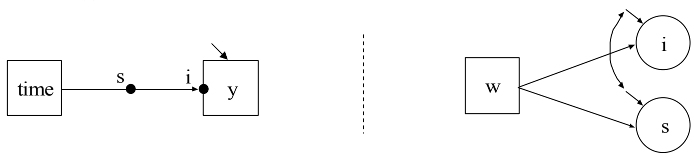


$$y_{ti} = i_i + s_i \times \text{time}_{ti} + \varepsilon_{ti}$$

i_i regressed on w_i
s_i regressed on w_i

- Long: Univariate, 2-Level Approach (CLUSTER = id)

Within Between



The intercept i is called y in Mplus

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Multilevel Modeling In A Latent Variable Framework

Integrating multilevel and SEM analyses (Asparouhov & Muthén, 2002).

Flexible combination of random effects and other latent variables:

- Multilevel models with random effects (intercepts, slopes)
  - Individually-varying times of observation read as data
  - Random slopes for time-varying covariates
- SEM with factors on individual and cluster levels
- Models combining random effects and factors, e.g.
  - Cluster-level latent variable predictors with multiple indicators
  - Individual-level latent variable predictors with multiple indicators
- Special applications
  - Random coefficient regression (no clustering; heteroscedasticity)
  - Interactions between continuous latent variables and observed variables

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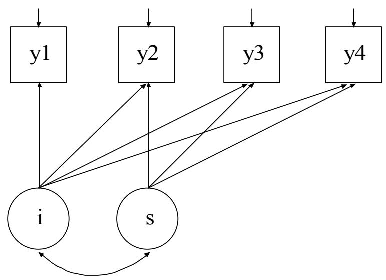
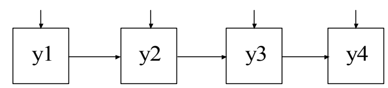
Alternative Models For Longitudinal Data

Growth Curve Model



Auto-Regressive Model



Hybrid Models

Curran & Bollen (2001)
McArdle & Hamagami (2001)

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Advantages Of Growth Modeling In A Latent Variable Framework

- Flexible curve shape
- Individually-varying times of observation
- Regressions among random effects
- Multiple processes
- Modeling of zeroes
- Multiple populations
- Multiple indicators
- Embedded growth models
- Categorical latent variables: growth mixtures

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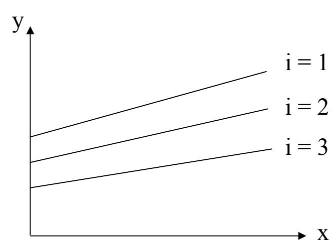
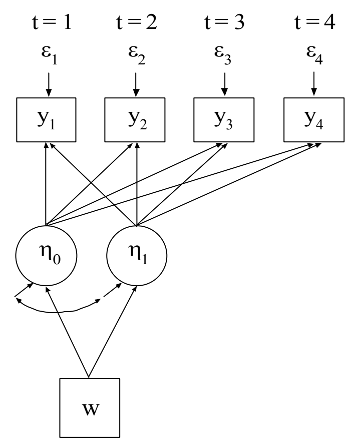
The Latent Variable Growth Model In Practice

Individual Development Over Time





(1) $y_{ti} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti}$

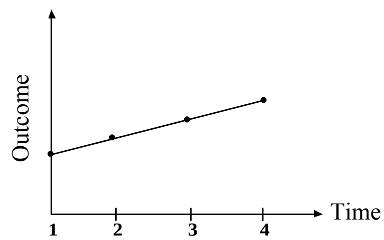
(2a) $\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}$

(2b) $\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}$

Specifying Time Scores For Linear Growth Models

Linear Growth Model

- Need two latent variables to describe a linear growth model: Intercept and slope



- Equidistant time scores for slope:

Equidistant time scores	0	1	2	3
for slope:	0	.1	.2	.3

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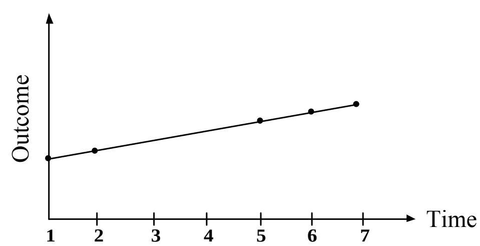
Specifying Time Scores For Linear Growth Models (Continued)



Nonequidistant time scores	0	1	4	5	6
for slope:	0	.1	.4	.5	.6

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Interpretation Of The Linear Growth Factors

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti}, \qquad (17)$$

where in the example t = 1, 2, 3, 4 and x_t = 0, 1, 2, 3:

$$y_{1i} = \eta_{0i} + \eta_{1i}\, 0 + \varepsilon_{1i}, \qquad (18)$$
$$\eta_{0i} = y_{1i} - \varepsilon_{1i}, \qquad (19)$$
$$y_{2i} = \eta_{0i} + \eta_{1i}\, 1 + \varepsilon_{2i}, \qquad (20)$$
$$y_{3i} = \eta_{0i} + \eta_{1i}\, 2 + \varepsilon_{3i}, \qquad (21)$$
$$y_{4i} = \eta_{0i} + \eta_{1i}\, 3 + \varepsilon_{4i}. \qquad (22)$$

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Interpretation Of The Linear Growth Factors (Continued)

Interpretation of the intercept growth factor

η_{0i} (initial status, level):
Systematic part of the variation in the outcome variable at the time point where the time score is zero.

- Unit factor loadings

Interpretation of the slope growth factor

η_{1i} (growth rate, trend):
Systematic part of the increase in the outcome variable for a time score increase of one unit.

- Time scores determined by the growth curve shape

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Interpreting Growth Model Parameters

- Intercept Growth Factor Parameters
  - Mean
    - Average of the outcome over individuals at the timepoint with the time score of zero;
    - When the first time score is zero, it is the intercept of the average growth curve, also called initial status
  - Variance
    - Variance of the outcome over individuals at the timepoint with the time score of zero, excluding the residual variance

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Interpreting Growth Model Parameters (Continued)

- Linear Slope Growth Factor Parameters
  - Mean – average growth rate over individuals
  - Variance – variance of the growth rate over individuals
  - Covariance with Intercept – relationship between individual intercept and slope values
- Outcome Parameters
  - Intercepts – not estimated in the growth model – fixed at zero to represent measurement invariance
  - Residual Variances – time-specific and measurement error variation
  - Residual Covariances – relationships between time-specific and measurement error sources of variation across time

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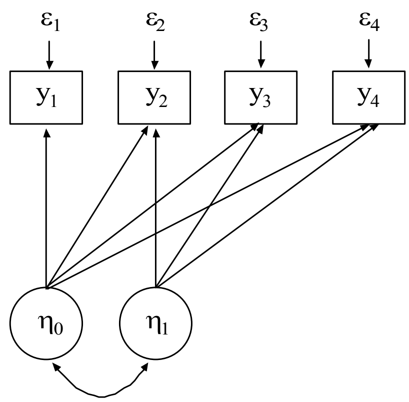
Latent Growth Model Parameters And Sources Of Model Misfit



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Latent Growth Model Parameters For Four Time Points

Linear growth over four time points, no covariates.

Free parameters in the H_1 unrestricted model:

- 4 means and 10 variances-covariances

Free parameters in the H_0 growth model:

(9 parameters, 5 d.f.):

- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:

- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

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Latent Growth Model Sources Of Misfit

Sources of misfit:
- Time scores for slope growth factor
- Residual covariances for outcomes
- Outcome variable intercepts
- Loadings for intercept growth factor

Model modifications:
- Recommended
  - Time scores for slope growth factor
  - Residual covariances for outcomes
- Not recommended
  - Outcome variable intercepts
  - Loadings for intercept growth factor

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Latent Growth Model Parameters For Three Time Points

Linear growth over three time points, no covariates.

Free parameters in the H_1 unrestricted model:
- 3 means and 6 variances-covariances

Free parameters in the H_0 growth model
(8 parameters, 1 d.f.)
- Means of intercept and slope growth factors
- Variances of intercept and slope growth factors
- Covariance of intercept and slope growth factors
- Residual variances for outcomes

Fixed parameters in the H_0 growth model:
- Intercepts of outcomes at zero
- Loadings for intercept growth factor at one
- Loadings for slope growth factor at time scores
- Residual covariances for outcomes at zero

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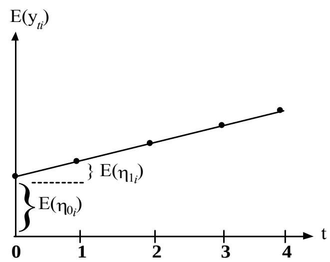
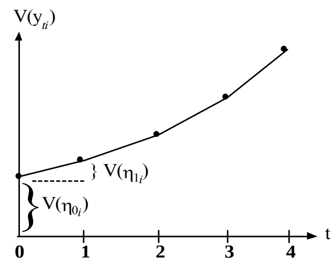
Growth Model Means And Variances

$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,,$$
$$x_t = 0,\, 1,\, \ldots,\, T - 1.$$

Expectation (mean; E) and variance (V):

$$E\,(y_{ti}) = E\,(\eta_{0i}\,) + E\,(\eta_{1i})\; x_t\,,$$
$$V\,(y_{ti}) = V\,(\eta_{0i}\,) + V\,(\eta_{1i})\; x_t^2$$
$$+\, 2x_t\, Cov\,(\eta_{0i}\,,\, \eta_{1i}) + \mathrm{V}\,(\varepsilon_{ti})$$





$V(\varepsilon_{ti})$ constant over t
$Cov(\eta_0\,,\, \eta_1) = 0$

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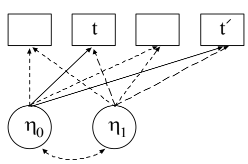
Growth Model Covariances

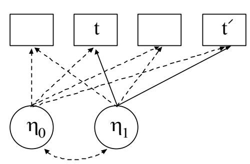
$$y_{ti} = \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,,$$
$$x_t = 0,\, 1,\, \ldots,\, T - 1.$$

$$Cov(y_{ti}, y_{t'i}) = V(\eta_{0i}) + V(\eta_{1i})\; x_t\, x_{t'}$$
$$+\, Cov(\eta_{0i}\,,\, \eta_{1i})\; (x_t + x_{t'})$$
$$+\, Cov(\varepsilon_{ti}\,,\, \varepsilon_{t'i}\,).$$

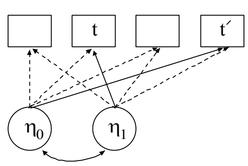
$V(\eta_{0i})$:



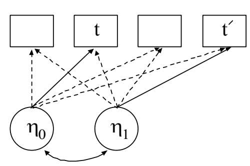
$V(\eta_{1i})\, x_t\, x_{t'}$:



$Cov(\eta_{0i}\,, \eta_{1i})\, x_t$:



$Cov(\eta_{0i}\,, \eta_{1i})\, x_{t'}$:



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Growth Model Estimation, Testing, And Model Modification

- Estimation: Model parameters
  - Maximum-likelihood (ML) estimation under normality
  - ML and non-normality robust s.e.'s
  - Quasi-ML (MUML): clustered data (multilevel)
  - WLS: categorical outcomes
  - ML-EM: missing data, mixtures
- Model Testing
  - Likelihood-ratio chi-square testing; robust chi square
  - Root mean square of approximation (RMSEA): Close fit ($\leq .05$)
- Model Modification
  - Expected drop in chi-square, EPC
- Estimation: Individual growth factor values (factor scores)
  - Regression method – Bayes modal – Empirical Bayes
  - Factor determinacy

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Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

$$y_{ti} = \mathbf{0} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \qquad (32)$$

$$\eta_{0i} = \boldsymbol{\alpha}_0 + \zeta_{0i}\,, \qquad (33)$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \qquad (34)$$

Parameterization 2 – for categorical outcomes and multiple indicators

$$y_{ti} = \boldsymbol{\nu} + \eta_{0i} + \eta_{1i}\, x_t + \varepsilon_{ti}\,, \qquad (35)$$

$$\eta_{0i} = \mathbf{0} + \zeta_{0i}\,, \qquad (36)$$

$$\eta_{1i} = \alpha_1 + \zeta_{1i}\,. \qquad (37)$$

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Alternative Growth Model Parameterizations

Parameterization 1 – for continuous outcomes

- Outcome variable intercepts fixed at zero
- Growth factor means free to be estimated

```
MODEL:   i BY y1-y4@1;
         s BY y1@0 y2@1 y3@2 y4@3;
         [y1-y4@0 i s];
```

Parameterization 2 – for categorical outcomes and multiple indicators

- Outcome variable intercepts constrained to be equal
- Intercept growth factor mean fixed at zero

```
MODEL:   i BY y1-y4@1;
         s BY y1@0 y2@1 y3@2 y4@3;
         [y1-y4] (1);
         [i@0 s];
```

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Simple Examples Of Growth Modeling

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Steps In Growth Modeling

- Preliminary descriptive studies of the data: means, variances, correlations, univariate and bivariate distributions, outliers, etc.
- Determine the shape of the growth curve from theory and/or data
    - Individual plots
    - Mean plot
- Consider change in variance across time
- Fit model without covariates using fixed time scores
- Modify model as needed
- Add covariates

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LSAY Data

The data come from the Longitudinal Study of American Youth (LSAY). Two cohorts were measured at four time points beginning in 1987. Cohort 1 was measured in Grades 10, 11, and 12. Cohort 2 was measured in Grades 7, 8, 9, and 10. Each cohort contains approximately 60 schools with approximately 60 students per school. The variables measured include math and science achievement items, math and science attitude measures, and background information from parents, teachers, and school principals. There are approximately 60 items per test with partial item overlap across grades – adaptive tests.

Data for the analysis include the younger females. The variables include math achievement from Grades 7, 8, 9, and 10 and the background variables of mother's education and home resources.

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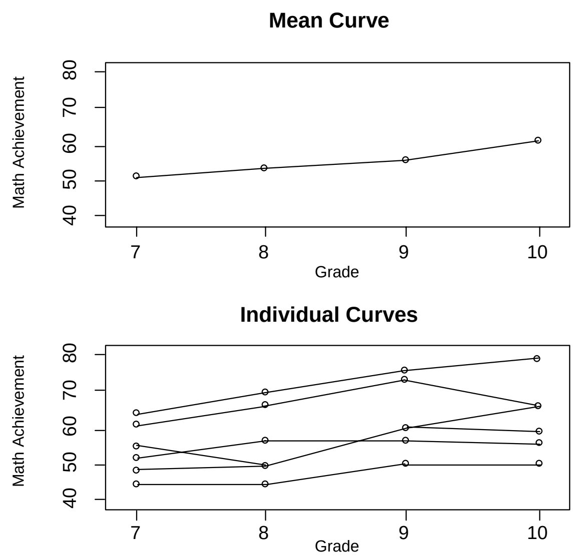

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Input For LSAY TYPE=BASIC Analysis

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           TYPE=BASIC Analysis
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = BASIC;
PLOT:      TYPE = PLOT1;
```

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Sample Statistics For LSAY Data

n = 984

Means

	MATH7	MATH8	MATH9	MATH10
	52.750	55.411	59.128	61.796

Covariances

	MATH7	MATH8	MATH9	MATH10
MATH7	81.107			
MATH8	67.663	82.829		
MATH9	73.150	76.513	100.986	
MATH10	77.952	82.668	95.158	131.326

Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.000			
MATH8	0.826	1.000		
MATH9	0.808	0.837	1.000	
MATH10	0.755	0.793	0.826	1.000

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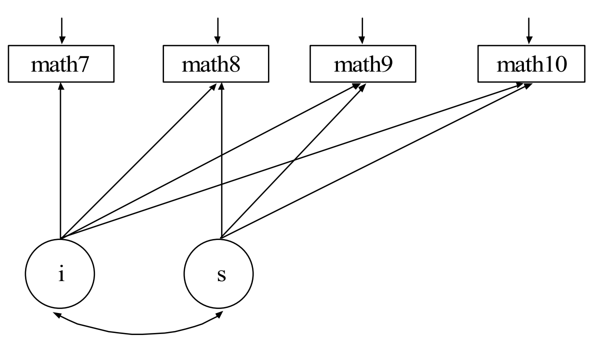

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Input For LSAY Linear Growth Model Without Covariates

```
TITLE:     LSAY For Younger Females With Listwise Deletion
           Linear Growth Model Without Covariates
DATA:      FILE IS lsay.dat;
           FORMAT IS 3F8.0 F8.4 8F8.2 3F8.0;
VARIABLE:  NAMES ARE cohort id school weight math7 math8 math9
           math10 att7 att8 att9 att10 gender mothed homeres;
           USEOBS = (gender EQ 1 AND cohort EQ 2);
           MISSING = ALL (999);
           USEVAR = math7-math10;
ANALYSIS:  TYPE = MEANSTRUCTURE;
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9@2 math10@3;
           [math7-math10@0];
           [i s];
OUTPUT:    SAMPSTAT STANDARDIZED MODINDICES (3.84);
```

Alternative language:

```
MODEL: i s | math7@0 math8@1 math9@2 math10@3;
```

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Output Excerpts LSAY Linear Growth Model Without Covariates

Tests Of Model Fit

```
Chi-Square Test of Model Fit
          Value                        22.664
          Degrees of Freedom                5
          P-Value                      0.0004

CFI/TLI
          CFI                          0.995
          TLI                          0.994

RMSEA (Root Mean Square Error Of Approximation)
          Estimate                     0.060
          90 Percent C.I.              0.036   0.086
          Probability RMSEA <= .05     0.223

SRMR (Standardized Root Mean Square Residual)
          Value                        0.025
```

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Modification Indices

	M.I.	E.P.C.	Std.E.P.C.	StdYX E.P.C.
S BY MATH7	6.793	0.185	0.254	0.029
S BY MATH8	14.694	-0.169	-0.233	-0.025
S BY MATH9	9.766	0.155	0.213	0.021

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	Std	StdYX
I BY					
MATH7	1.000	.000	.000	8.029	.906
MATH8	1.000	.000	.000	8.029	.861
MATH9	1.000	.000	.000	8.029	.800
MATH10	1.000	.000	.000	8.029	.708
S BY					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.377	.148
MATH9	2.000	.000	.000	2.753	.274
MATH10	3.000	.000	.000	4.130	.364

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

Means					
I	52.623	.275	191.076	6.554	6.554
S	3.105	.075	41.210	2.255	2.255
Intercepts					
MATH7	.000	.000	.000	.000	.000
MATH8	.000	.000	.000	.000	.000
MATH9	.000	.000	.000	.000	.000
MATH10	.000	.000	.000	.000	.000

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Output Excerpts LSAY Linear Growth Model Without Covariates (Continued)

I WITH					
S	3.491	.730	4.780	.316	.316
Residual Variances					
MATH7	14.105	1.253	11.259	14.105	.180
MATH8	13.525	.866	15.610	13.525	.156
MATH9	14.726	.989	14.897	14.726	.146
MATH10	25.989	1.870	13.898	25.989	.202
Variances					
I	64.469	3.428	18.809	1.000	1.000
S	1.895	.322	5.894	1.000	1.000

R-Square

Observed Variable	R-Square
MATH7	0.820
MATH8	0.844
MATH9	0.854
MATH10	0.798

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Growth Model With Free Time Scores

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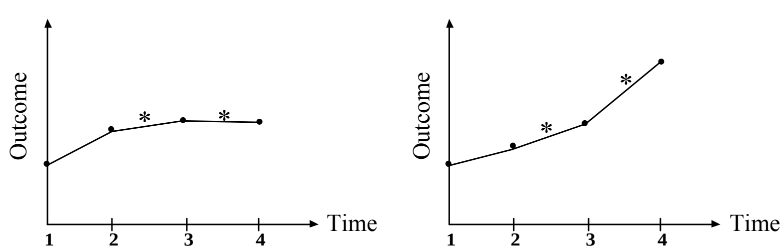
Specifying Time Scores For Non-Linear Growth Models With Estimated Time Scores

Non-linear growth models with estimated time scores

- Need two latent variables to describe a non-linear growth model: Intercept and slope



Time scores: 0 1 Estimated Estimated

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Interpretation Of Slope Growth Factor Mean For Non-Linear Models

- The slope growth factor mean is the change in the outcome variable for a one unit change in the time score
- In non-linear growth models, the time scores should be chosen so that a one unit change occurs between timepoints of substantive interest.
  - An example of 4 timepoints representing grades 7, 8, 9, and 10
    - Time scores of 0 1 * * – slope factor mean refers to change between grades 7 and 8
    - Time scores of 0 * * 1 – slope factor mean refers to change between grades 7 and 10

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Growth Model With Free Time Scores

- Identification of the model – for a model with two growth factors, at least one time score must be fixed to a non-zero value (usually one) in addition to the time score that is fixed at zero (centering point)
- Interpretation—cannot interpret the mean of the slope growth factor as a constant rate of change over all timepoints, but as the rate of change for a time score change of one.
- Approach—fix the time score following the centering point at one
- Choice of time score starting values if needed

• Means	52.75		55.41		59.13		61.80
• Differences		2.66		3.72		2.67	
• Time scores	0		1		>2		>2+1

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Input Excerpts For LSAY Linear Growth Model With Free Time Scores Without Covariates

```
MODEL:     i s | math7@0 math8@1 math9 math10;

OUTPUT:    RESIDUAL;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7@0 math8@1 math9 math10;
           [math7-math10@0];
           [i s];
```

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates

n = 984

Tests Of Model Fit

```
Chi-Square Test of Model Fit
              Value                                4.222
              Degrees of Freedom                       3
              P-Value                             0.2373
CFI/TLI
              CFI                                  1.000
              TLI                                  0.999
RMSEA (Root Mean Square Error Of Approximation)
              Estimate                             0.020
              90 Percent C.I.                      0.000   0.061
              Probability RMSEA <= .05             0.864
SRMR (Standardized Root Mean Square Residual)
              Value                                0.015
```

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I |					
MATH7	1.000	.000	.000	8.029	.903
MATH8	1.000	.000	.000	8.029	.870
MATH9	1.000	.000	.000	8.029	.797
MATH10	1.000	.000	.000	8.029	.708
S |					
MATH7	.000	.000	.000	.000	.000
MATH8	1.000	.000	.000	1.134	.123
MATH9	**2.452**	**.133**	**18.442**	**2.780**	**.276**
MATH10	**3.497**	**.199**	**17.540**	**3.966**	**.350**

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

	Estimates	S.E.	Est./S.E.	Std	StdYX
S WITH					
I	3.110	.600	5.186	.342	.342
Variances					
I	64.470	3.394	18.994	1.000	1.000
S	**1.286**	**.265**	**4.853**	**1.000**	**1.000**
Means					
I	52.785	.283	186.605	6.574	6.574
S	**2.586**	**.167**	**15.486**	**2.280**	**2.280**

Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Residuals

Model Estimated Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
52.785	55.370	59.123	61.827

Residuals for Means/Intercepts/Thresholds

MATH7	MATH8	MATH9	MATH10
-.035	.041	.004	-.031

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Output Excerpts LSAY Growth Model With Free Time Scores Without Covariates (Continued)

Model Estimated Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	79.025			
MATH8	67.580	85.180		
MATH9	72.094	78.356	101.588	
MATH10	75.346	82.952	93.994	128.477

Residuals for Covariances/Correlations/Residual Correlations

	MATH7	MATH8	MATH9	MATH10
MATH7	1.999			
MATH8	.014	-2.436		
MATH9	.981	-1.921	-.705	
MATH10	2.527	-.368	1.067	2.715

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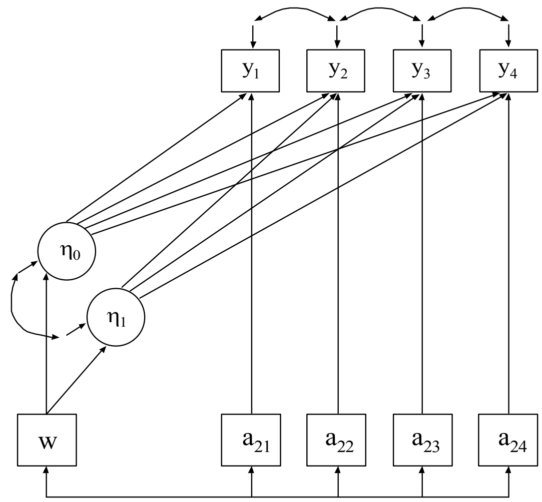
Covariates In The Growth Model

Covariates In The Growth Model

- Types of covariates
  - Time-invariant covariates—vary across individuals not time, explain the variation in the growth factors
  - Time-varying covariates—vary across individuals and time, explain the variation in the outcomes beyond the growth factors

Time-Invariant And Time-Varying Covariates



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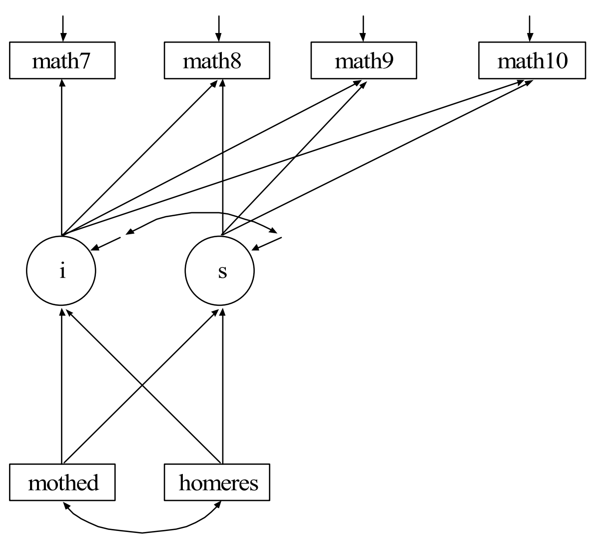
LSAY Growth Model With Time-Invariant Covariates



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Input Excerpts For LSAY Linear Growth Model With Free Time Scores And Covariates

```
VARIABLE:   NAMES ARE cohort id school weight math7 math8 math9
            math10 att7 att8 att9 att10 gender mothed homeres;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 mothed homeres;
ANALYSIS:   !ESTIMATOR = MLM;
MODEL:      i s | math7@0 math8@1 math9 math10;
            i s ON mothed homeres;
```

Alternative language:

```
MODEL:      i BY math7-math10@1;
            s BY math7@0 math8@1 math9 math10;
            [math7-math10@0];
            [i s];
            i s ON mothed homeres;
```


Output Excerpts LSAY Growth Model With Free Time Scores And Covariates

n = 935

Tests Of Model Fit for ML

```
Chi-Square Test of Model Fit
             Value                                 15.845
             Degrees of Freedom                         7
             P-Value                               0.0265
CFI/TLI
             CFI                                    0.998
             TLI                                    0.995
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                               0.037
             90 Percent C.I.                        0.012  0.061
             Probability RMSEA <= .05               0.794
SRMR (Standardized Root Mean Square Residual)
             Value                                  0.015
```

Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Tests Of Model Fit for MLM

```
Chi-Square Test of Model Fit
             Value                                8.554*
             Degrees of Freedom                       7
             P-Value                             0.2862
             Scaling Correction Factor            1.852
                 for MLM
CFI/TLI
             CFI                                  0.999
             TLI                                  0.999
RMSEA (Root Mean Square Error Of Approximation)
             Estimate                             0.015
SRMR (Standardized Root Mean Square Residual)
             Value                                0.015
WRMR (Weighted Root Mean Square Residual)
             Value                                0.567
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

Selected Estimates For ML

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	**2.054**	**.281**	**7.322**	**.257**	**.247**
HOMERES	**1.376**	**.182**	**7.546**	**.172**	**.255**
S ON					
MOTHED	.103	.068	1.524	.094	.090
HOMERES	**.149**	**.045**	**3.334**	**.136**	**.201**
I WITH					
S	2.604	.559	4.658	.297	.297
Residual Variances					
I	53.931	2.995	18.008	.842	.842
S	1.134	.253	4.488	.942	.942
Intercepts					
I	43.877	.790	55.531	5.484	5.484
S	1.859	.221	8.398	1.695	1.695

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates (Continued)

R-Square

```
Observed
Variable      R-Square

MATH7         0.813
MATH8         0.849
MATH9         0.861
MATH10        0.796

Latent
Variable      R-Square

I             .158
S             .058
```


Model Estimated Average And Individual Growth Curves With Covariates

Model:

$$y_{ti} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{ti}, \qquad (23)$$

$$\eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}, \qquad (24)$$

$$\eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}, \qquad (25)$$

Estimated growth factor means:

$$\hat{E}(\eta_{0i}) = \hat{\alpha}_0 + \hat{\gamma}_0 \overline{w}, \qquad (26)$$

$$\hat{E}(\eta_{1i}) = \hat{\alpha}_1 + \hat{\gamma}_1 \overline{w}\ . \qquad (27)$$

Estimated outcome means:

$$\hat{E}(y_{ti}) = \hat{E}(\eta_{0i}) + \hat{E}(\eta_{1i})\, x_t\ . \qquad (28)$$

Estimated outcomes for individual *i* :

$$\hat{y}_{ti} = \hat{\eta}_{0i} + \hat{\eta}_{1i}\ x_t \qquad (29)$$

where $\hat{\eta}_{0i}$ and $\hat{\eta}_{1i}$ are estimated factor scores. $\hat{y}_{ti}$ can be used for prediction purposes.

Model Estimated Means With Covariates

Model estimated means are available using the TECH4 and RESIDUAL options of the OUTPUT command.

Estimated Intercept Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

43.88 + 2.05*2.31 + 1.38*3.11 = 52.9

Estimated Slope Mean = Estimated Intercept +
Estimated Slope (Mothed)*
Sample Mean (Mothed) +
Estimated Slope (Homeres)*
Sample Mean (Homeres)

1.86 + .10*2.31 + .15*3.11 = 2.56

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Model Estimated Means With Covariates (Continued)

Estimated Outcome Mean at Timepoint t =

Estimated Intercept Mean +
Estimated Slope Mean * (Time Score at Timepoint t)

Estimated Outcome Mean at Timepoint 1 =
52.9 + 2.56 * (0) = **52.9**
Estimated Outcome Mean at Timepoint 2 =
52.9 + 2.56 * (1.00) = **55.46**
Estimated Outcome Mean at Timepoint 3 =
52.9 + 2.56 * (2.45) = **59.17**
Estimated Outcome Mean at Timepoint 4 =
52.9 + 2.56 * (3.50) = **61.86**

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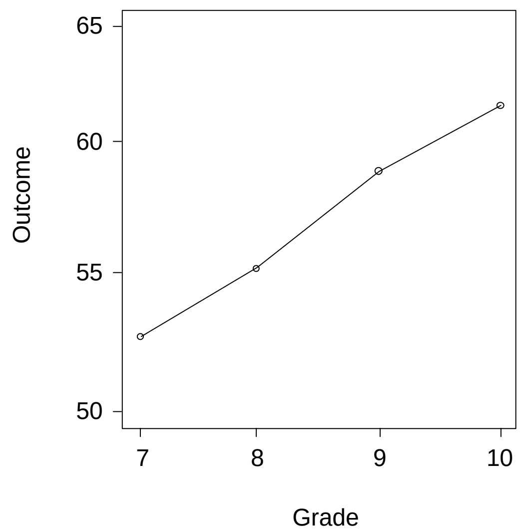
Estimated LSAY Curve



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Centering

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Centering

- Centering determines the interpretation of the intercept growth factor
- The centering point is the timepoint at which the time score is zero
- A model can be estimated for different centering points depending on which interpretation is of interest
- Models with different centering points give the same model fit because they are reparameterizations of the model
- Changing the centering point in a linear growth model with four timepoints

Timepoints	1	2	3	4	Centering at
Time scores	0	1	2	3	Timepoint 1
	-1	0	1	2	Timepoint 2
	-2	-1	0	1	Timepoint 3
	-3	-2	-1	0	Timepoint 4

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Input Excerpts For LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

```
MODEL:     i s | math7*-3 math8*-2 math9@-1 math10@0;
           i s ON mothed homeres;
```

Alternative language:

```
MODEL:     i BY math7-math10@1;
           s BY math7*-3 math8*-2 math9@-1 math10@0;
           [math7-math10@0];
           [i s];
           i s ON mothed homeres;
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10

n = 935

Tests of Model Fit

```
CHI-SQUARE TEST OF MODEL FIT

          Value                          15.845
          Degrees of Freedom                  7
          P-Value                         .0265

RMSEA (ROOT MEAN SQUARE ERROR OF APPROXIMATION)

          Estimate                        .037
          90 Percent C.I.                 .012    .061
          Probability RMSEA <= .05        .794
```

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Output Excerpts LSAY Growth Model With Free Time Scores And Covariates Centered At Grade 10 (Continued)

Selected Estimates

	Estimates	S.E.	Est./S.E.	Std	StdYX
I ON					
MOTHED	2.418	0.353	6.851	0.238	0.229
HOMERES	1.903	0.229	8.294	0.187	0.277
S ON					
MOTHED	0.111	0.073	1.521	0.094	0.090
HOMERES	0.161	0.049	3.311	0.136	0.201

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Further Readings On Introductory Growth Modeling

Bijleveld, C. C. J. H., & van der Kamp, T. (1998). Longitudinal data analysis: Designs, models, and methods. Newbury Park: Sage.

Muthén, B. & Khoo, S.T. (1998). Longitudinal studies of achievement growth using latent variable modeling. Learning and Individual Differences, Special issue: latent growth curve analysis, 10, 73-101. (#80)

Muthén, B. & Muthén, L. (2000). The development of heavy drinking and alcohol-related problems from ages 18 to 37 in a U.S. national sample. Journal of Studies on Alcohol, 61, 290-300. (#83)

Raudenbush, S.W. & Bryk, A.S. (2002). Hierarchical linear models: Applications and data analysis methods. Second edition. Newbury Park, CA: Sage Publications.

Singer, J.D. & Willett, J.B. (2003). Applied longitudinal data analysis. Modeling change and event occurrence. New York, NY: Oxford University Press.

Snijders, T. & Bosker, R. (1999). Multilevel analysis. An introduction to basic and advanced multilevel modeling. Thousand Oakes, CA: Sage Publications.

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