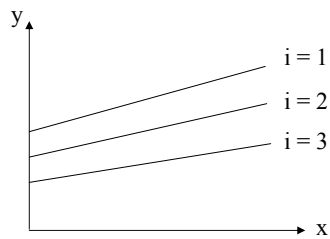


Growth Mixture Modeling

1

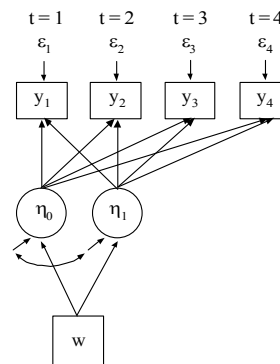
Individual Development Over Time



$$(1) \quad y_{it} = \eta_{0i} + \eta_{1i} x_t + \varepsilon_{it}$$

$$(2a) \quad \eta_{0i} = \alpha_0 + \gamma_0 w_i + \zeta_{0i}$$

$$(2b) \quad \eta_{1i} = \alpha_1 + \gamma_1 w_i + \zeta_{1i}$$



2

Mixtures and Latent Trajectory Classes

Modeling motivated by substantive theories of:

- Multiple Disease Processes: Prostate cancer (Pearson et al.)
- Multiple Pathways of Development: Adolescent-limited versus life-course persistent antisocial behavior (Moffitt), crime curves (Nagin), alcohol development (Zucker, Schulenberg)
- Subtypes: Subtypes of alcoholism (Cloninger, Zucker)

3

Example: Mixed-Effects Regression Models For Studying The Natural History Of Prostate Disease

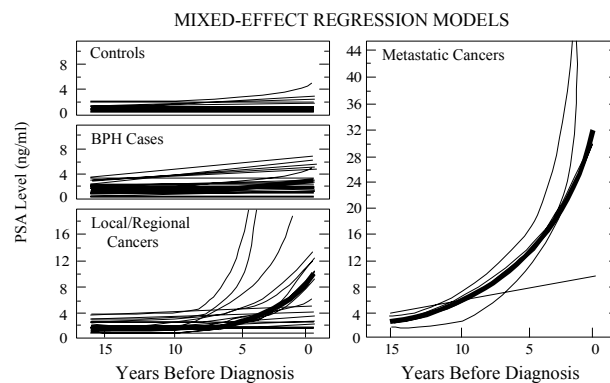
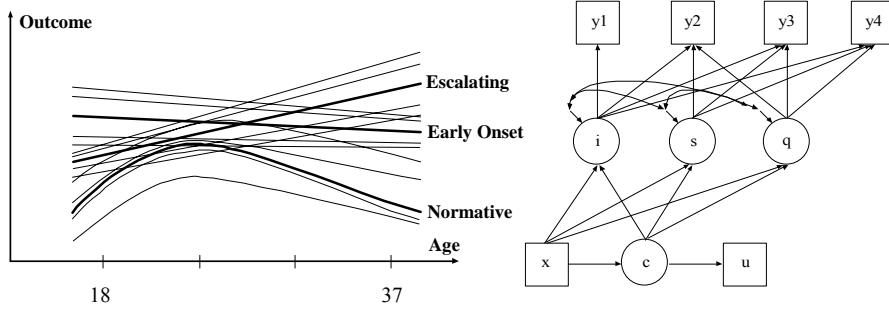


Figure 2. Longitudinal PSA curves estimated from the linear mixed-effects model for the group average (thick solid line) and for each individual in the study (thin solid lines)

Source: Pearson, Morrell, Landis and Carter (1994), Statistics in Medicine

4

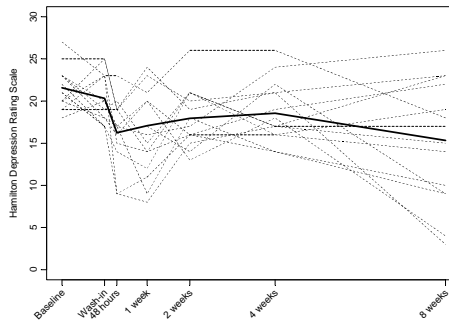
Growth Mixture Modeling Of Developmental Pathways



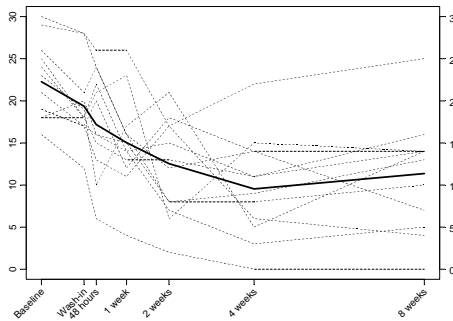
5

A Clinical Trial Of Depression Medication: Two-Class Growth Mixture Modeling

Placebo Non-Responders, 55%



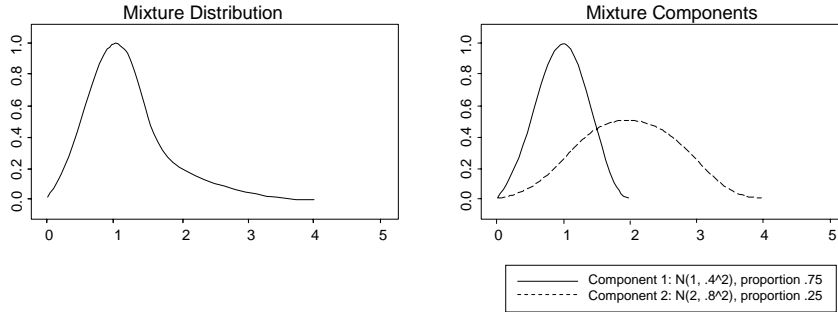
Placebo Responders, 45%



6

Mixture Distributions

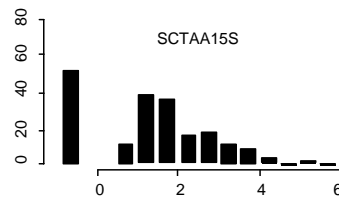
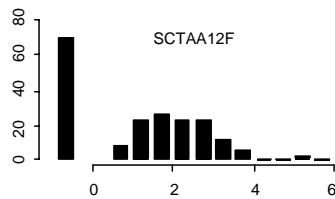
Non-normality for mixture, normality for mixture components



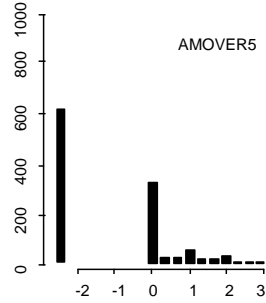
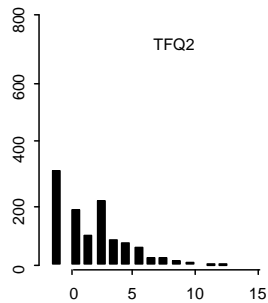
7

Two Types Of Distributions

(1) Normal mixture components



(2) Preponderance of zeroes



8

Growth Mixture Analysis

Generalization of conventional random effect growth modeling (multilevel modeling) to include qualitatively different developments (Muthén & Shedden, 1999 in Biometrics).
Combination of conventional growth modeling and cluster analysis (finite mixture analysis).

- Setting
 - Longitudinal data
 - A single or multiple items measured repeatedly
 - Hypothesized trajectory classes (categorical latent variable)
 - Individual trajectory variation within classes

9

Growth Mixture Analysis (Continued)

- Aim
 - Estimate trajectory shapes
 - Estimate trajectory class probabilities
 - Estimate variation within class
 - Relate class probabilities to covariates
 - Relate within-class variation to covariates
 - Classify individuals into classes (posterior prob's)

Application: Mathematics achievement, grades 7 – 10 (LSAY)
related to mother's education and home resources.
National sample, $n = 846$.

10

Strategies For Finding The Number Of Classes In Growth Mixture Modeling

- **Comparing models with different numbers of classes**
 - BIC – low BIC value corresponds to a high loglikelihood value and a parsimonious model
 - TECH11 – Lo-Mendell-Rubin likelihood ratio test (Biometrika, 2001)
 - TECH14 – bootstrapped LRT (Version 4)
- **Residuals and model tests**
 - TECH7 – class-specific comparisons of model-estimated means, variances, and covariances versus posterior probability-weighted sample statistics
 - TECH12 – class-mixed residuals for univariate skewness and kurtosis
 - TECH13 – multivariate skew and kurtosis model tests

11

Strategies For Finding The Number Of Classes In Growth Mixture Modeling (Continued)

- **Classification quality**
 - Posterior probabilities – classification table and entropy
 - Individual trajectory classification using pseudo classes (Bandeem-Roche et al., 1997; Muthén et al. in Biostatistics, 2002)
- **Interpretability and usefulness of the latent classes**
 - Trajectory shapes
 - Number of individuals in each class
 - Number of estimated parameters
 - Substantive theory
 - Auxiliary (external) variables – predictive validity

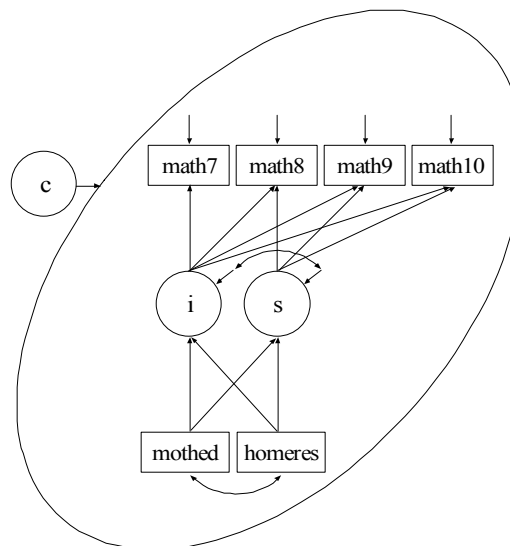
12

Strategies For Finding Starting Values In Growth Mixture Modeling (Continued)

- **Strategy 1**
 - Do a conventional growth analysis (one-class model)
 - Use estimated growth factor means and standard deviations as growth factor mean starting values in a multi-class model – mean plus and minus .5 standard deviation
- **Strategy 2**
 - Estimate a multi-class model with the variances and covariances of the growth factors fixed to zero (LCGA)
 - Use the estimated growth factor means as growth factor mean starting values for a model with growth factor variances and covariances free

Random starts makes it unnecessary to give starting values: starts = 50 5

13



14

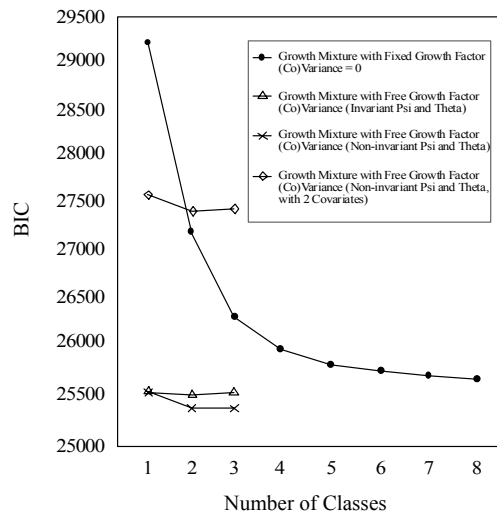
Deciding On The Number Of Classes For The LSAY Growth Mixture Model

n = 935

Number of Classes	1	2	3
Loglikelihood	-11,997.653	-11,864.826	-11,856.220
# parameters	15	29	36
BIC	24,098	23,928	23,959
AIC	24,025	23,788	23,784
Entropy	NA	.468	.474
TECH11 LRT p-value for k-1 classes	NA	.0000	.4041
Multivariate skew p-value	.00	.34	.26
Multivariate kurtosis p-value	.00	.10	.05

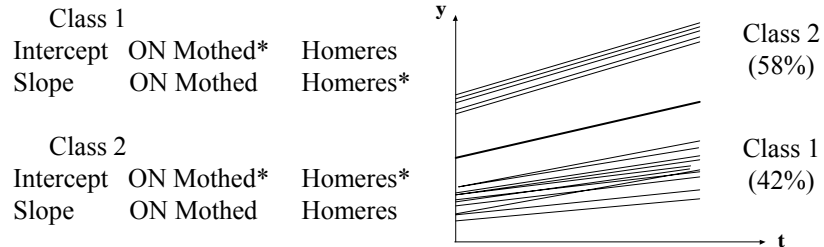
15

Model Fit by BIC: LSAY



16

LSAY: Estimated Two-Class Growth Mixture Model



Conventional Single-Class Analysis

Intercept	ON Mothed*	Homeres*
Slope	ON Mothed	Homeres*

17

Input For LSAY Two-Class Growth Mixture Model

```

TITLE:      2-class varying slopes on mothed and homeres
            varying Psi varying Theta

DATA:      FILE IS lsay.dat;
            FORMAT IS 3f8 f8.4 8f8.2 2f8.2;

VARIABLE:  NAMES ARE cohort id school weight math7
            math8 math9 math10 att7 att8 att9 att10 gender mothed
            homeres;
            USEOBS = (gender EQ 1 AND cohort EQ 2);
            MISSING = ALL (999);
            USEVAR = math7-math10 mothed homeres;
            CLASSES = c(2);

ANALYSIS:  TYPE = MIXTURE;
    
```

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Input For LSAY Two-Class Growth Mixture Model (Continued)

```
MODEL:      %OVERALL%
            intercept slope | math7@0 math8@1 math9 math10;
            intercept slope ON mothed homeres;
            %c#2%
            intercept slope ON mothed homeres;
            math7-math10 intercept slope;
            slope WITH intercept;

OUTPUT:     TECH8 TECH12 TECH13 RESIDUAL;
```

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Output Excerpts LSAY Two-Class Growth Mixture Model

Tests Of Model Fit

Loglikelihood		
	H0 Value	-11864.825
Information Criteria		
	Number of Free Parameters	29
	Akaike (AIC)	23787.649
	Bayesian (BIC)	23928.025
	Sample-Size Adjusted BIC ($n^* = (n + 2) / 24$)	23835.923
	Entropy	0.468

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Classification Information

FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE

Class 1	392.19327	0.41946
Class 2	542.80673	0.58054

CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS MEMBERSHIP

Class Counts and Proportions

Class 1	342	0.36578
Class 2	593	0.63422

Average Class Probabilities by Class

	1	2
Class 1	0.853	0.147
Class 2	0.170	0.830

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

TECHNICAL 12 OUTPUT

ESTIMATED MIXED MODEL AND RESIDUALS (OBSERVED - EXPECTED)

Observed Skewness

<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>	<u>HOMERES</u>
-0.184	-0.312	-0.349	-0.471	1.214	-0.087

Estimated Mixed Skewness

<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>	<u>HOMERES</u>
-0.160	-0.190	-0.348	-0.517	0.002	-0.012

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

TECHNICAL 13 OUTPUT

SKEW AND KURTOSIS TESTS OF MODEL FIT

TWO-SIDED MULTIVARIATE SKEW TEST OF FIT

Sample Value	1.245
Mean	0.999
Standard Deviation	0.275
P-Value	0.3400

TWO-SIDED MULTIVARIATE KURTOSIS TEST OF FIT

Sample Value	29.612
Mean	27.842
Standard Deviation	1.015
P-Value	0.1000

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Model Results

	Estimates	S.E.	Est./S.E.	
Class 1				
INTERCPT				
MATH7	1.000	.000	.000	
MATH8	1.000	.000	.000	
MATH9	1.000	.000	.000	
MATH10	1.000	.000	.000	
SLOPE				
MATH7	.000	.000	.000	
MATH8	1.000	.000	.000	
MATH9	2.422	.133	18.157	
MATH10	3.580	.204	17.570	
INTERCPT ON				
MOTHEd	1.656	.626	2.645	
HOMERES	.720	.377	1.911	
SLOPE ON				
MOTHEd	.146	.154	.953	
HOMERES	.228	.087	2.626	24

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.
SLOPE WITH INTERCPT	.727	1.643	.443
Residual Variances			
MATH7	17.198	2.840	6.055
MATH8	15.257	2.077	7.347
MATH9	24.170	3.294	7.337
MATH10	49.112	10.037	4.893
INTERCPT	54.297	6.094	8.910
SLOPE	1.643	.627	2.620
Intercepts			
MATH7	.000	.000	.000
MATH8	.000	.000	.000
MATH9	.000	.000	.000
MATH10	.000	.000	.000
INTERCPT	42.733	1.648	25.922
SLOPE	.816	.366	2.228

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.
Class 2			
INTERCPT			
MATH7	1.000	.000	.000
MATH8	1.000	.000	.000
MATH9	1.000	.000	.000
MATH10	1.000	.000	.000
SLOPE			
MATH7	.000	.000	.000
MATH8	1.000	.000	.000
MATH9	2.422	.133	18.157
MATH10	3.580	.204	17.570
INTERCPT ON			
MOTHEd	2.085	.376	5.545
HOMERES	1.805	.285	6.334
SLOPE ON			
MOTHEd	.028	.079	.358
HOMERES	.054	.058	.943

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Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	Estimates	S.E.	Est./S.E.	
SLOPE WITH INTERCPT	.478	.575	.831	
Residual Variances				
MATH7	12.248	1.587	7.720	
MATH8	11.375	1.244	9.141	
MATH9	8.014	1.420	5.642	
MATH10	7.349	1.312	5.600	
INTERCPT	32.431	4.651	6.973	
SLOPE	.191	.264	.724	
Intercepts				
MATH7	.000	.000	.000	
MATH8	.000	.000	.000	
MATH9	.000	.000	.000	
MATH10	.000	.000	.000	
INTERCPT	45.365	1.461	31.054	
SLOPE	2.847	.316	9.015	
LATENT CLASS REGRESSION MODEL PART				
Means C#1	-.325	.306	-1.063	27

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

Residuals

ESTIMATED MODEL AND RESIDUALS (OBSERVED - ESTIMATED) FOR CLASS 1

Model Estimated Means

<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>	<u>HOMERES</u>
48.650	50.486	53.095	55.221	2.255	3.030

Residuals for Means

<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>	<u>HOMERES</u>
-0.113	0.072	0.236	-0.389	0.000	0.000

Model Estimated Covariances

	<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>	
MATH7	76.058					
MATH8	60.370	78.945				
MATH9	62.515	68.408	100.958			
MATH10	64.263	72.253	83.615	141.979		
MOTHEd	1.751	1.962	2.263	2.507	0.907	28
HOMERES	2.309	2.908	3.760	4.454	0.345	

Output Excerpts LSAY Two-Class Growth Mixture Model (Continued)

	<u>HOMERES</u>				
HOMERES	2.412				
Residuals for Covariances					
	<u>MATH7</u>	<u>MATH8</u>	<u>MATH9</u>	<u>MATH10</u>	<u>MOTHEd</u>
MATH7	-0.153				
MATH8	-0.109	-0.143			
MATH9	0.413	0.572	0.451		
MATH10	-0.701	-0.614	-0.254	-0.338	
MOTHEd	0.210	-0.252	-0.021	0.256	0.000
HOMERES	0.289	-0.254	-0.358	0.720	0.000
	<u>HOMERES</u>				
HOMERES	0.000				

29

Further Readings On Growth Mixture Modeling

- Muthén, B. (2001). Second-generation structural equation modeling with a combination of categorical and continuous latent variables: New opportunities for latent class/latent growth modeling. In Collins, L.M. & Sayer, A. (Eds.), New methods for the analysis of change (pp. 291-322). Washington, D.C.: APA. (#82)
- Muthén, B. (2001). Latent variable mixture modeling. In G. A. Marcoulides & R. E. Schumacker (eds.), New developments and techniques in structural equation modeling (pp. 1-33). Lawrence Erlbaum Associates. (#86)
- Muthén, B. (2002). Beyond SEM: General latent variable modeling. Behaviormetrika, 29, 81-117. (#96)
- Muthén, B. (2004). Latent variable analysis: Growth mixture modeling and related techniques for longitudinal data. In D. Kaplan (ed.), Handbook of quantitative methodology for the social sciences (pp. 345-368). Newbury Park, CA: Sage Publications. (#100)

30

Further Readings On Growth Mixture Modeling (Continued)

Muthén, B. & Muthén, L. (2000). Integrating person-centered and variable-centered analysis: growth mixture modeling with latent trajectory classes. Alcoholism: Clinical and Experimental Research, 24, 882-891. (#85)

Muthén, B. & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469. (#78)

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General Growth Mixture Modeling

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General Growth Mixture Modeling (GGMM)

GGMM goes beyond conventional random effect growth modeling by using latent trajectory classes which

- Allow for heterogeneity with respect to
 - Growth functions – different classes correspond to different growth shapes
 - Antecedents – different background variables have different importance for different classes
 - Consequences – class membership predicts later outcomes
- Allow for prediction of trajectory class membership
- Allow for confirmatory clustering
 - With respect to parameters – describing curve shapes
 - With respect to typical individuals – known classes
- Allow for classification of individuals
 - Early prediction of problematic development
- Allow for enhanced preventive intervention analysis
 - Different classes benefit differently and can receive different treatments

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General Growth Mixture Modeling

Muthén (2001), Muthén & Muthén (2000), Muthén (2004)

- Setting
 - Longitudinal data
 - A single or multiple items measured repeatedly
 - Hypothesized trajectory classes (categorical latent variable)
 - Individual trajectory variation within classes
 - Prediction of distal outcomes
- Aim
 - Estimate trajectory shapes
 - Estimate trajectory class probabilities
 - Estimate variation within class
 - Relate class probabilities to covariates
 - Relate within-class variation to covariates
 - Relate class membership to distal outcomes
 - Classify individuals into classes (posterior prob's)

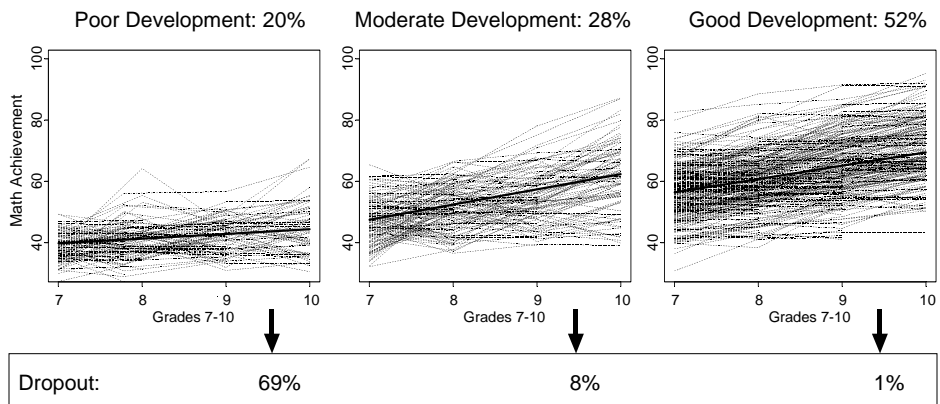
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General Growth Mixture Modeling (Continued)

- Applications
 - LSAY math achievement development and high school dropout
 - The development of heavy drinking ages 18-30 (NLSY) related to antecedents and consequences. National sample, n = 922

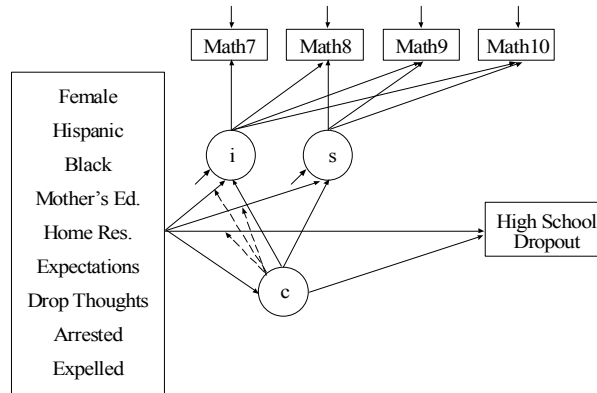
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Mplus Graphics For LSAY Math Achievement Trajectory Classes



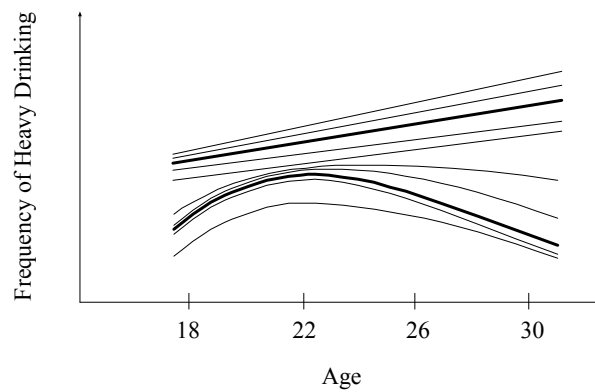
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LSAY Math Achievement Trajectory Classes



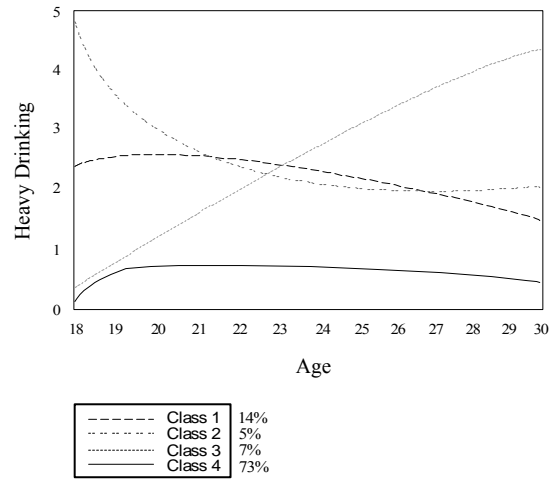
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Example: NLSY Heavy Drinking Two Latent Trajectory Classes



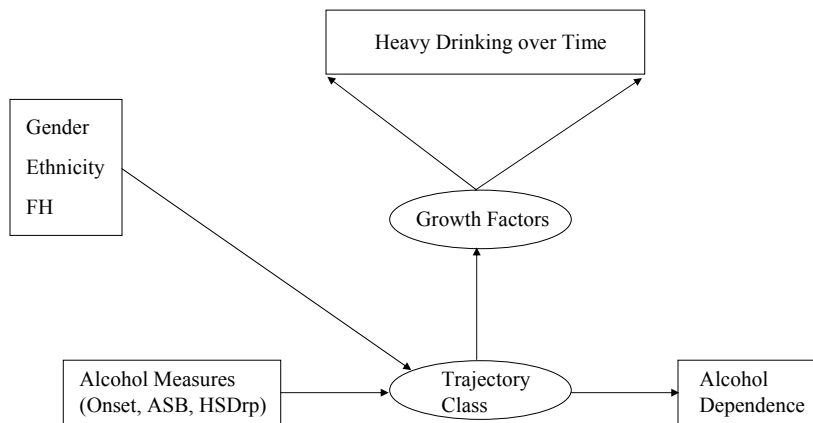
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NLSY: Heavy Drinking, Cohort 64



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NLSY: Antecedents And Consequences



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Multinomial Logistic Regression Of c ON x

The multinomial logistic regression model expresses the probability that individual i falls in class k of the latent class variable c as a function of the covariate x ,

$$P(c_j = k | x_i) = \frac{e^{\alpha_k + \gamma_k x_i}}{\sum_{s=1}^K e^{\alpha_s + \gamma_s x_i}}, \quad (90)$$

where $\alpha_K = 0, \gamma_K = 0$ so that $e^{\alpha_K + \gamma_K x_i} = 1$.

This implies that the log odds comparing class k to the last class K is

$$\log[P(c_i = k | x_i)/P(c_i = K | x_i)] = \alpha_k + \gamma_k x_i. \quad (91)$$

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Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922)

HD Class on Covariates						
	HD Classes					
	1 (Down)		2 (High18)		3 (Up)	
	Est.	t	Est.	t	Est.	t
Male	1.21	5.52	1.25	3.48	1.45	4.73
Black	-0.89	-3.43	-3.14	-2.86	-0.06	-0.17
Hisp	-0.65	-2.22	-0.35	-0.86	-0.01	-0.03
ES	1.24	4.79	2.05	5.72	0.71	1.78
FH1	0.03	0.09	-0.21	-0.41	-0.08	-0.16
FH23	0.04	0.15	0.25	0.56	0.08	0.23
FH123	-0.23	-0.58	1.18	2.59	1.00	2.60
HSDRP	0.57	1.98	0.32	0.76	0.91	2.93
Coll	-0.07	-0.31	-1.31	-2.85	-1.08	-2.59

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Heavy Drinking And Alcohol Dependence NLSY Cohort64 (n=922) (Continued)

Alcohol Dependence as a Function of Heavy Drinking Class

	Probability	Odds Ratio
HD Class 1 (Down)	0.16	3.92
HD Class 2 (High 18)	0.26	7.06
HD Class 3 (Up)	0.60	30.00
HD Class 4 (Norm)	0.05	1.00

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Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

```

TITLE:      NLSY for cohort 64 quadratic growth mixture model with
            covariates centered at 25: four-class model of heavy
            drinking with classes predicting dep94

VARIABLE:   CLASSES = c(4);
            CATEGORICAL IS dep94;

ANALYSIS:   TYPE = MIXTURE;

MODEL:      %OVERALL%
            c#1-c#3 ON male black hisp es fh1 fh23 fh123 hsdrrp coll;
            i s1 s2 | hd82@-3.008 hd83@-2.197 hd84@-1.621 hd88@-.235
                    hd89@.000 hd94@.884;
    
```

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Input Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

```

!      log age scale: x_t = a*(ln(t-b) - ln(c-b));
!      where t is time, a and b are constants to fit the mean curve
!      (chosen as a = 2 and b = 16), and c is the centering age,
!      here set at 25.

%c#1%                                ! Not needed
[dep94$1*1 i*2 s1*-.5 s2*-.1];      ! Not needed
%c#2%                                ! Not needed
[dep94$1*0 i*1 s1*-.2 s2*-.3];      ! Not needed
%c#3%                                ! Not needed
[dep94$1*.6 i*3 s1*1.5 s2*.2];      ! Not needed
%c#4%                                ! Not needed
[dep94$1*2 i*.6 s1*-.2 s2*-.1];    ! Not needed

```

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Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome

C#1	ON			
MALE		1.214	.220	5.515
BLACK		-.886	.258	-3.434
HISP		-.645	.290	-2.223
ES		1.240	.259	4.789
FH1		.026	.291	.088
FH23		.039	.261	.149
FH123		-.233	.399	-.583
HSDRP		.566	.286	1.976
COLL		-.071	.231	-.308

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**Output Excerpts NLSY Growth Mixture Model
With Covariates And A Distal Outcome (Continued)**

C#2	ON				
	MALE	1.248	.359	3.481	
	BLACK	-3.138	1.097	-2.860	
	HISP	-.346	.401	-.864	
	ES	2.045	.357	5.722	
	FH1	-.211	.514	-.410	
	FH23	.247	.444	.556	
	FH123	1.178	.456	2.585	
	HSDRP	.323	.428	.756	
	COLL	-1.311	.460	-2.851	
C#3	ON				
	MALE	1.454	.308	4.727	
	BLACK	-.059	.344	-.171	
	HISP	-.011	.369	-.030	
	ES	.712	.399	1.784	
	FH1	-.079	.502	-.157	
	FH23	.084	.364	.232	
	FH123	1.004	.387	2.596	
	HSDRP	.913	.312	2.926	
	COLL	-1.075	.414	-2.594	47

**Output Excerpts NLSY Growth Mixture Model
With Covariates And A Distal Outcome (Continued)**

Class 1				
Thresholds				
DEP94\$1	1.631	0.248	6.574	
Class 2				
Thresholds				
DEP94\$1	1.041	0.338	3.077	
Class 3				
Thresholds				
DEP94\$1	-0.406	0.272	-1.493	
Class 4				
Thresholds				
DEP94\$1	2.987	0.208	14.392	

Output Excerpts NLSY Growth Mixture Model With Covariates And A Distal Outcome (Continued)

Classification Information

FINAL CLASS COUNTS AND PROPORTIONS OF TOTAL SAMPLE SIZE

Class 1	135.95653	0.14746
Class 2	45.86689	0.04975
Class 3	74.68767	0.08101
Class 4	665.48891	0.72179

CLASSIFICATION OF INDIVIDUALS BASED ON THEIR MOST LIKELY CLASS MEMBERSHIP

Class Counts and Proportions

Class 1	134	0.14534
Class 2	46	0.04989
Class 3	72	0.07809
Class 4	670	0.72668

Average Class Probabilities by Class

	1	2	3	4
Class 1	0.994	0.000	0.000	0.005
Class 2	0.003	0.997	0.000	0.000
Class 3	0.007	0.000	0.947	0.047
Class 4	0.003	0.000	0.010	0.987

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General Growth Mixture Modeling With Sequential Processes

- New setting:
 - Sequential, linked processes
- New aims:
 - Using an earlier process to predict a later process
 - Early prediction of failing class

Application: General growth mixture modeling of first- and second-grade reading skills and their Kindergarten precursors; prediction of reading failure (Muthén, Khoo, Francis, Boscardin, 1999). Suburban sample, n = 410.

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Assessment of Reading Skills Development

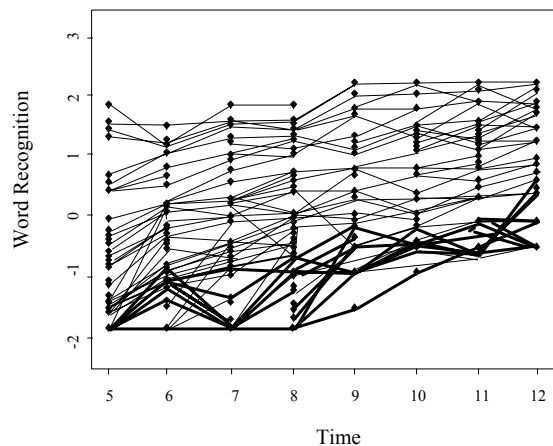
- Longitudinal multiple-cohort design involving approximately 1000 children with measurements taken four times a year from Kindergarten through grade two (October, December, February, April)
- Grade 1 – Grade 2: reading and spelling skills
- Precursor skills: phonemic awareness (Kindergarten, Grade 1, Grade 2), letters/names/sounds (Kindergarten only), rapid naming
- Standardized reading comprehension tests at the end of Grade 1 and Grade 2 (May).

Three research hypotheses (EARS study; Francis, 1996):

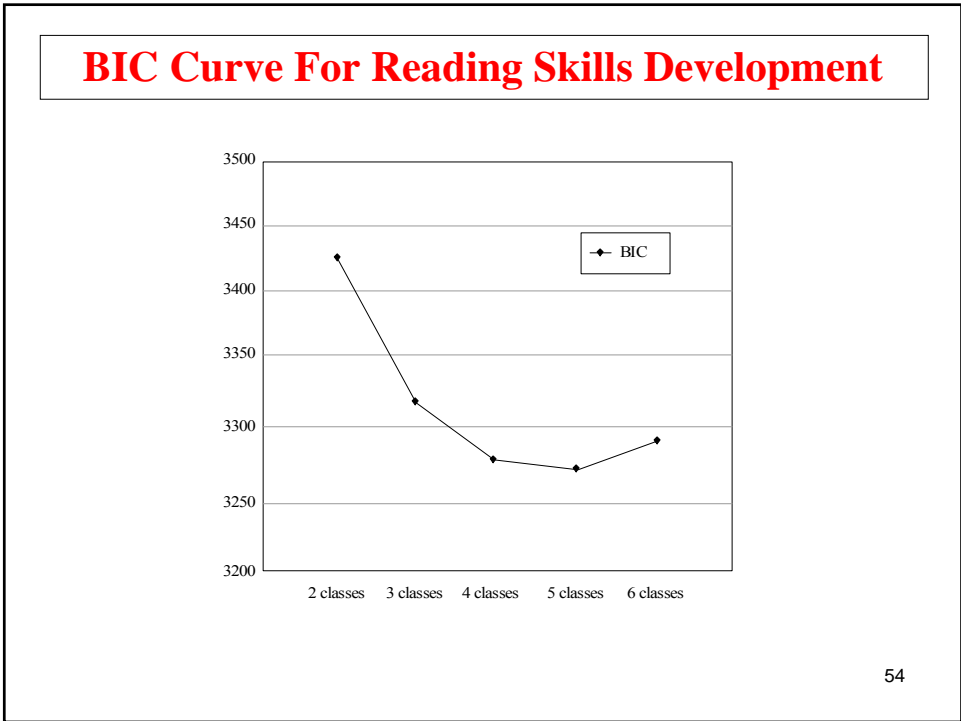
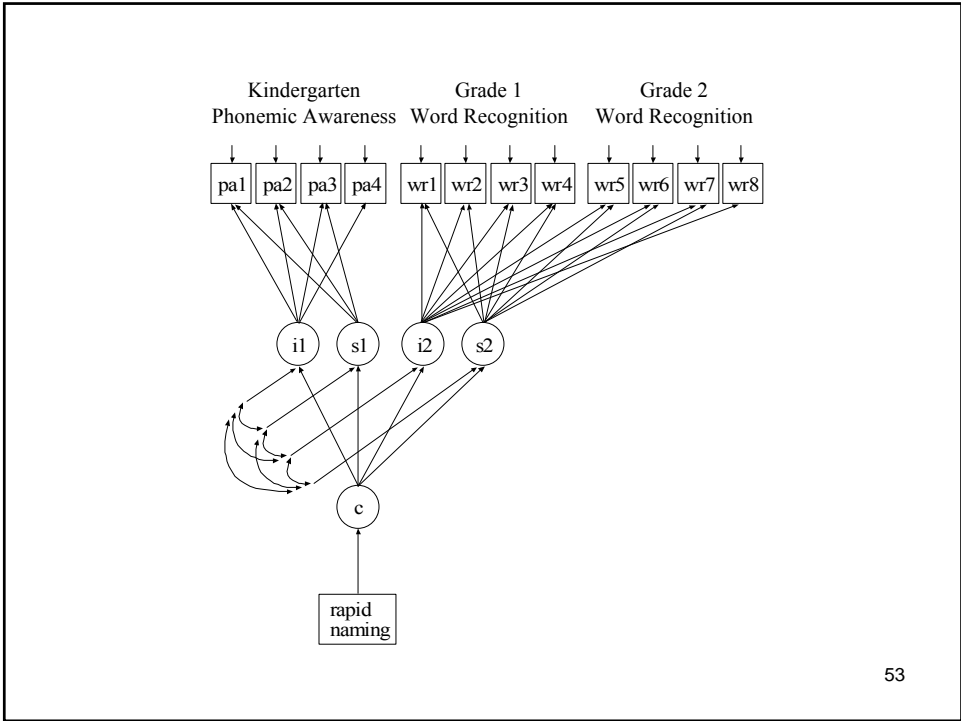
- Kindergarten children will differ in their growth and development in precursor skills
- The rate of development of the precursor skills will relate to the rate of development and the level of attainment of reading and spelling skills – and the individual growth rates in reading and spelling skills will predict performance on standardized tests of reading and spelling
- The use of growth rates for skills and precursors will allow for earlier identification of children at risk for poor academic outcomes and lead to more stable predictions regarding future academic performance

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Word Recognition Development In Grades 1 And 2



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Input For Growth Mixture Model For Reading Skills Development

```

TITLE:      Growth mixture model for reading skills development

DATA:      FILE IS newran.dat;

VARIABLE:  NAMES ARE gender eth wc pa1-pa4 wr1-wr8 l1-l4 s1 r1 s2 r2
           rnam1g1 rnam1g2 rnam1g3 rnam1g4;
           USEVAR = pa1-wr8 rnam1g4;
           MISSING ARE ALL (999);
           CLASSES = c(5);

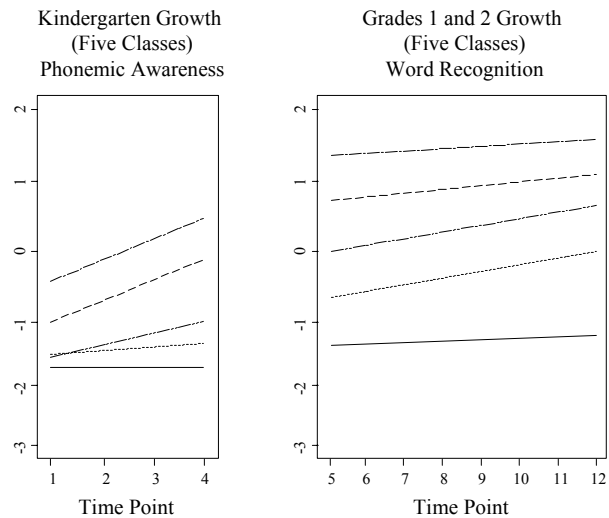
ANALYSIS:  TYPE = MIXTURE MISSING;

MODEL:     %OVERALL%
           i1 s1 | pa1@-3 pa2@-2 pa3@-1 pa4@0;
           i2 s2 | wr1@-7 wr2@-6 wr3@-5 wr4@-4 wr5@-3 wr6@-2
           wr7@-1 wr8@0;
           c#1-c#4 ON rnam1g4;

OUTPUT:    TECH8;
    
```

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Five Classes Of Reading Skills Development



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How Early Can A Good Classification Be Made?

Focus on Class 1, the failing class.

1. Estimate full growth mixture model for Kindergarten, Grade 1, and Grade 2 outcomes
2. Use the estimated full model to classify students into classes based on the posterior probabilities for each class, where a student is classified into the class with the largest posterior probability.
3. Classify students using early information by holding parameters fixed at the estimates from the full model of Step 1 and classifying individuals using Kindergarten information only, adding Grade 1 outcomes, adding Grade 2 outcomes
4. Study quality of early classification by cross-tabulating individuals classified as in Steps 2 and 3 (sensitivity and specificity)

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Sensitivity And Specificity Of Early Classification

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K Only	1.00	28	7	3			38
	2.00	10	29	16			55
	3.00	8	33	100	25		166
	4.00		1	24	63	17	105
	5.00		1	1	12	32	46
Total		46	71	144	100	49	410

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 1 Only	1.00	28	7	3			38
	2.00	15	44	24			83
	3.00	3	20	112	20		155
	4.00			5	79	4	88
	5.00			1	1	45	46
Total		46	71	144	100	49	410

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 2 Only	1.00	28	8				36
	2.00	16	54	22			92
	3.00	2	9	119	7		137
	4.00			4	91	4	99
	5.00				2	45	47
Total		46	71	145	100	49	411

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Sensitivity And Specificity Of Early Classification (Continued)

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 3 Only	1.00	37	12				49
	2.00	9	53	8			70
	3.00		6	136	4		146
	4.00			1	95	1	97
	5.00				1	48	49
Total		46	71	145	100	49	411

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 4 Only	1.00	45	11				56
	2.00	1	57	3			61
	3.00		3	141	2		146
	4.00			1	97		98
	5.00				1	49	50
Total		46	71	145	100	49	411

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 5 Only	1.00	45	3				48
	2.00	1	66				67
	3.00		2	145	1		148
	4.00				98		98
	5.00				1	49	50
Total		46	71	145	100	49	411

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Sensitivity And Specificity Of Early Classification (Continued)

		Full Model					Total
		1.00	2.00	3.00	4.00	5.00	
K + 6 Only	1.00	46					46
	2.00		69				69
	3.00		1	145	1		147
	4.00				98		98
	5.00				1	49	50
Total		46	70	145	100	49	410

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Growth Mixtures In Randomized Trials

Different treatment effects in different trajectory classes

Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P. Kellam, S., Carlin, J., & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. *Biostatistics*, 3, 459-475.

61

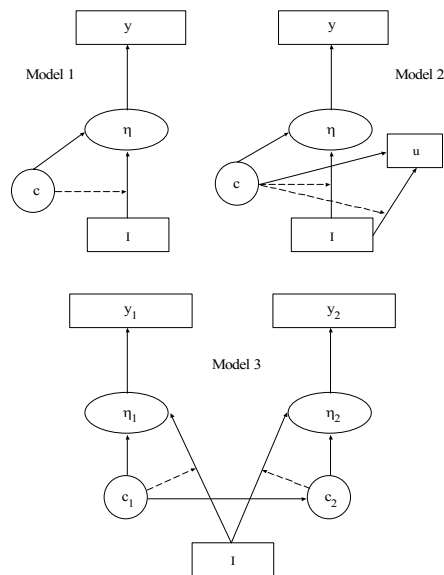
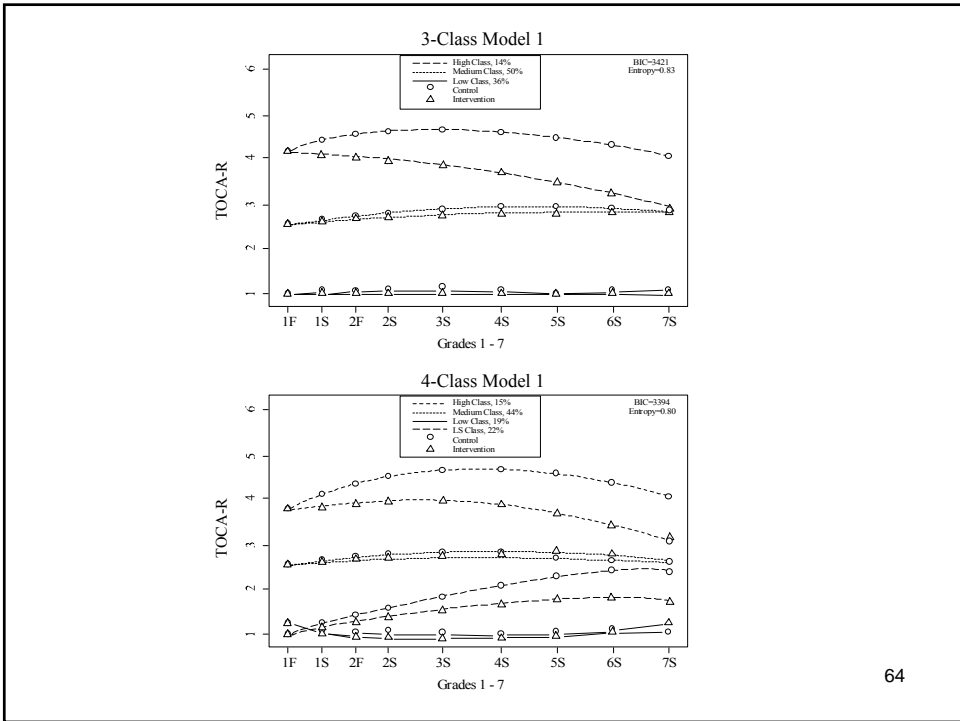
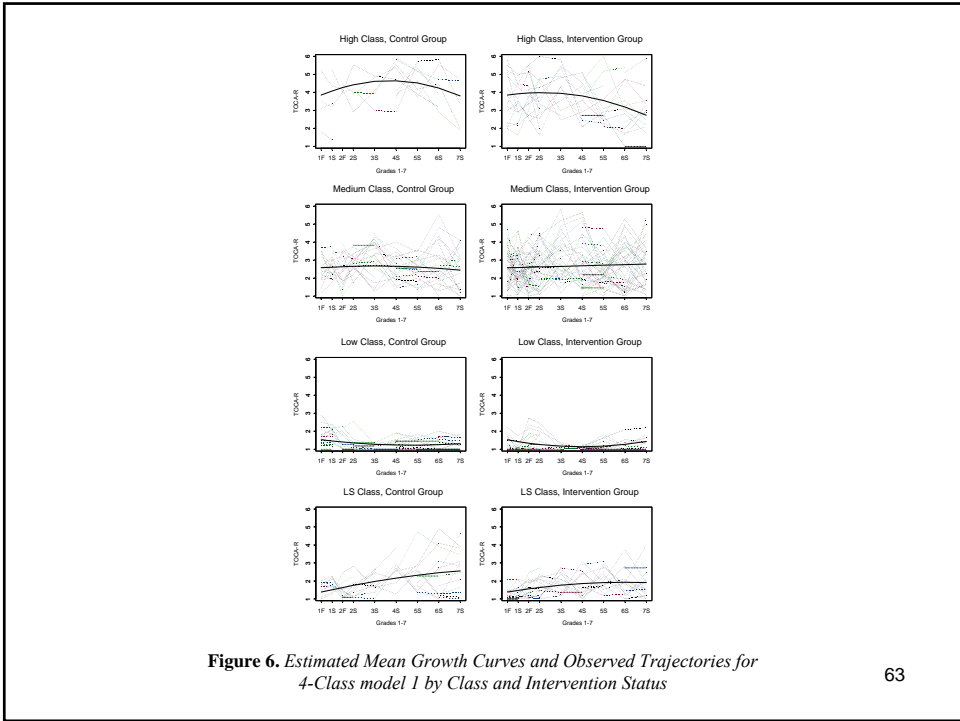
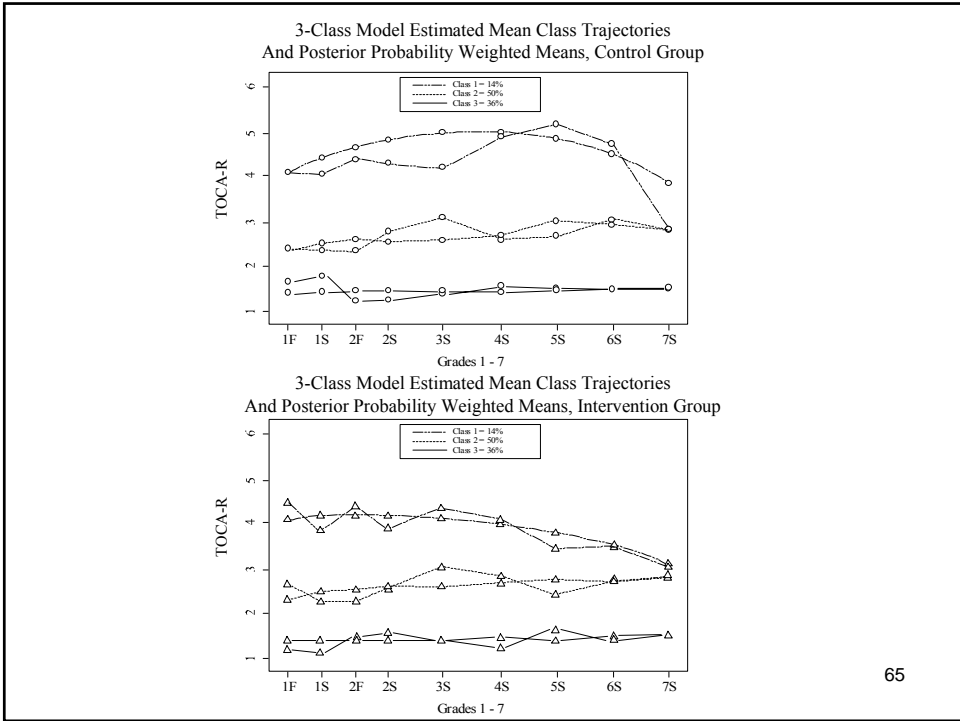


Figure 1. Path Diagrams for Models 1 - 3

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Input For Growth Mixtures In Randomized Trials

```

TITLE:      growth mixtures in randomized trials
DATA:      FILE IS toca.dat;
VARIABLE:  NAMES ARE sctaa11f sctaa11s sctaa12f sctaa12s sctaa13s
            sctaa14s sctaa15s sctaa16s sctaa17s intngrp;
MISSING ARE ALL (999);
USEVARIABLES ARE sctaa11f-sctaa17s tx;
CLASSES = c(3);

DEFINE:    tx = (intngrp==4);
ANALYSIS:  TYPE = MIXTURE MISSING;
MODEL:    %OVERALL%
            ac bc qc | sctaa11f@0 sctaa11s@0.5 sctaa12f@1
            sctaa12s@1.5 sctaa13s@2.5 sctaa14s@3.5 sctaa15s@4.5
            sctaa16s@5.5 sctaa17s@6.5;
            qc@0;
            bc qc ON tx;
            sctaa11f WITH sctaa11s; sctaa12f WITH sctaa12s;
    
```

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Input For Growth Mixtures In Randomized Trials (Continued)

```
%c#1%  
[ac*3 bc qc]; bc qc ON tx;  
%c#2%  
[ac*2 bc qc]; bc qc ON tx;  
%c#3%  
[ac*1 bc qc]; bc qc ON tx;  
ac sctaa11f-sctaa17s;
```

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Further Readings On General Growth Mixture Modeling

- Muthén, B. (2004). Latent variable analysis: Growth mixture modeling and related techniques for longitudinal data. In D. Kaplan (ed.), Handbook of quantitative methodology for the social sciences (pp. 345-368). Newbury Park, CA: Sage Publications. (#100)
- Muthén, B. & Shedden, K. (1999). Finite mixture modeling with mixture outcomes using the EM algorithm. Biometrics, 55, 463-469. (#78)
- Muthén, B., Brown, C.H., Masyn, K., Jo, B., Khoo, S.T., Yang, C.C., Wang, C.P., Kellam, S., Carlin, J. & Liao, J. (2002). General growth mixture modeling for randomized preventive interventions. Biostatistics, 3, 459-475. (#87)
- Muthén, B., Khoo, S.T., Francis, D. & Kim Boscardin, C. (2002). Analysis of reading skills development from Kindergarten through first grade: An application of growth mixture modeling to sequential processes. In S.R. Reise & N. Duan (eds), Multilevel modeling: Methodological advances, issues, and applications (pp. 71 – 89). Mahaw, NJ: Lawrence Erlbaum Associates. (#77)

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