This article discusses how a factor model with continuous latent variables can be used to analyze a set of strongly skewed dichotomous items and how such a model can be used for classification of subjects. The suitability of the specification of normally distributed latent variables, as is assumed with the use of tetrachoric correlations, is investigated. Both exploratory and confirmatory analyses, including multiple groups with mean structures, are illustrated. Substantive findings include support for unidimensionality of the items used in the DSM-III diagnosis of depression and a large degree of invariance in factor structure for the Baltimore and Durham sites.

Dichotomous Factor Analysis of Symptom Data

BENGT O. MUTHÉN
University of California, Los Angeles

This article considers applications of psychometric models for factor analysis of dichotomously scored symptom data. The models to be used are extensions of the standard factor analysis model for continuous, interval-scaled variables, sharing the standard specification of continuous, interval-scaled factor variables. The main aim of the article is to use such modeling to address specific research questions for the ECA data from the Baltimore and Durham sites, data that have been described by Eaton and Bohrnstedt in the introductory chapter. Specifically, the factor structure of a set of 41 symptom items for approximately 7,000 individuals will be investigated with a view toward a classification of these individuals. A major research question is how much overlap there is of symptoms of anxiety and depression. Another question concerns the appropriateness of the classificatory structure of the Major Depressive Disorder as laid out in the current DSM-III diagnostic manual. Also, the similarity of the factor structure across the two sites is of interest.

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Although these analyses will be carried out on the ECA data, it should be recognized that they contain several methodological features that are of a more general psychometric nature. A major feature is how to best factor analyze dichotomous variables corresponding to very rare events. This calls into question the appropriateness of the standard model specification of symmetric, normal distributions for the underlying latent variables.

In the next section, general issues in the factor analysis of dichotomous variables will be reviewed briefly with a view toward the specific problem of the analysis of very rare events. In the third section, the factor analysis will be carried out for the Baltimore site, while the fourth section considers the Durham site. The final section summarizes.

METHODS

MOTIVATING THE BASIC MODEL

The factor analysis model may be described as follows: For a set of p response variables y^*, consider a linear model with m factors η

$$y^* = \Lambda\eta + \varepsilon \qquad [1]$$

where Λ is a p × m matrix of factor loadings and ε contains p residuals. With usual assumptions, the covariance structure for the y^*'s is

$$V(y^*) = \Lambda\Psi\Lambda' + \Theta \qquad [2]$$

where Ψ is the factor covariance matrix and Θ is the residual covariance matrix.

Because the factors of η and the residuals of ε are specified to be continuous, interval-scaled random variables, the y^*'s will be of the same scale. Consider now p measured symptom variables y, one y variable for each y^* variable. Assume that the y's are scored 0/1. A standard factor analysis would take $y = y^*$ for each of the p variables. This naturally presents an illogical model in which the right-hand side of (1) is a continuous, interval-scaled quantity while the left-hand side is a discrete dichotomous variable. Each of the p equations in (1) represents a linear regression of a y^* on a set of explanatory variables η. The problem of assuming $y = y^*$ is well known in the regression context and has been thoroughly discussed in terms of OLS regression versus logistic regression; for example, Aldrich and Nelson (1984). In this case, the error term cannot meet standard model assumptions, causing incorrect standard errors of parameter estimates.

The underlying problem is that a linear model is assumed for the relationship between each y and the η's. It is more appropriate to specify a linear model for the relationship between each y^* and the η's, with a nonlinear relationship between each y and the η's. The y^* variable for each observed symptom variable y is a latent response variable that may be thought of as follows. Assume for simplicity that the set of symptoms measures a single factor, depression. Assume that for a certain y^* the loading on this factor is positive. The y^* variable may then be thought of as the specific tendency to report a certain symptom. When the tendency exceeds a variable-specific threshold, the respondent reports the symptom, otherwise not. Using the parameter τ for this y^*, we write

$$y = \begin{cases} 0, & \text{if } y^* < \tau \\ 1, & \text{otherwise} \end{cases} \qquad [3]$$

This recognizes that not all respondents who report the existence of the symptom, with the observation y = 1, have experienced the same degree of severity of this symptom. Similarly, some of the respondents who answer y = 0 may in fact have a y^* value closely below the threshold while others are far below it. In this way, it is plausible that the y^* variables have a linear regression on η, fulfilling standard assumptions for the residual.

The relationship between y and y^* in (3) leads to a nonlinear relationship between y and η, expressing not the value of y but the probability of y as a function of η. This is appropriate because what is needed is a response model for a discrete y, the binomial distribution of which is described by probabilities. Assuming that the residual ε is independent of η and that it has a normal or logistic density result in the nonlinear relationship between y and η,

$$P(y = 0 \mid \eta) = P(Y^* < \tau) = F(\tau - \lambda\eta) \qquad [4]$$

where F is the standard normal or logistic distribution function and it is assumed that the residual variance is standardized to 1. The relationship in (4) is that of probit or logit regression and is the appropriate way to relate the symptoms to the factors. This is also the model used in Item Response Theory (IRT), also called latent trait theory (Lord and Novick, 1968; see Reiser, this issue as well), in which the influence of a single latent ability trait on a dichotomously scored multiple-choice test item is described by an item characteristic curve, sometimes with a third parameter added to the two parameters used in (4). IRT assumes conditional independence of the y's given the η, which in the y^* formulation corresponds to the regular factor analysis assumption of uncorrelated residuals ε.

Taking $y = y^*$ and using a standard (factor or regression) analysis leads to a linear model with estimates that are in fact biased and inconsistent estimates for the nonlinear model. In factor analysis this is often described in terms of which correlation coefficients should be analyzed for dichotomous variables. Choosing the linear model of $y = y^*$ implies the use of regular Pearson product-moment coefficients (phi coefficients), while specifying the nonlinear relationship of (3) and (4) leads to the use of tetrachoric correlations. It is well known in the psychometric literature (for examples, see Lord and Novick, 1968) that phi coefficients are not appropriate measures of association because their maxima are restricted by the two univariate distributions and therefore they are not free to vary between −1 and +1. The phi coefficients refer to the correlations between y variables and the tetrachoric coefficients refer to the correlations between y^* variables. Because the y^* variables are continuous and unlimited, these are the proper correlations to analyze. It is also well known that the phi coefficients are attenuated relative to the tetrachoric correlations and that the attenuation is a function of the univariate distributions. This fact has led to the classic factor analysis problem of "difficulty factors," in which it has been observed for test items that some resulting factors may be artifactual and merely correspond to items with similar distributions (for example, Carroll, 1961; Ferguson, 1941; Olsson, 1979). When, as in the present case, the items are all positively correlated and are all skewed in the same direction with a similar degree of skewness, the factor analysis of phi coefficients may not lead to much distortion in terms of the pattern of factor loadings. The loadings are, however, strongly attenuated and give a false impression of low variable reliability.

ESTIMATING THE MODEL

The factor analysis model for dichotomous variables may be estimated as follows. Arranging the sample correlations in one vector s, the limited-information generalized least squares (GLS) estimator may be written as

$$F = (s - \sigma)' W^{-1}(s - \sigma) \qquad [5]$$

where σ is the population counterpart of s, W is an estimate of the large-sample covariance matrix of s, and F is to be minimized with respect to the factor model parameters. The parameters enter into the elements of σ as shown in (2), where $V(y^*)$ contains the σ elements.

When W = I (the identity matrix), the unweighted least-squares (ULS) estimator that minimizes residual sums of squares is obtained. Using the GLS weight matrix, these residuals are weighted by their sampling variability, providing less variable estimates. GLS gives a direct way of testing model fit because with large samples a function of the minimum value of F is a chi-square variate with degrees of freedom equal to the number of model restrictions. The theory for this was provided in Muthén (1978), considering both exploratory and confirmatory (correlation structure) analyses. Muthén and Christoffersson (1981) generalized this to the simultaneous analysis of multiple groups with tests of across-group invariance hypotheses; for an application, see Muthén (1981). Muthén (1984) further extended this methodology to ordered polytomous variables and mixtures of categorical and continuous variables in general structural equation modeling settings. These methods and extensions thereof for censored variables are available in the software package LISCOMP (Muthén, 1987).

Mislevy (1986) gives an overview of various techniques for carrying out dichotomous factor analysis and also discusses maximum likelihood estimation, which uses full information instead of the limited information from lower-order marginal tables used by GLS. Experience has shown that the limited information approach gives results that are very similar to those of the optimal, full-information approach. Currently, the full-information approach is available only for exploratory factor analysis.

In the minimization of (5) it causes no problem if the sample tetrachoric correlation matrix is not positive definite (positive definiteness may here be loosely viewed as the multivariate counterpart of a positive variance estimate; regular Pearson product-moment correlation matrices are ensured positive definiteness as usually calculated, but tetrachorics are not because they are computed in a pairwise fashion). Although this has been advanced as a reason for not using tetrachorics, it would only be a problem when using such correlations in an analysis under the assumption of normally distributed observed variables requiring matrix inversion. A nonpositive definite tetrachoric correlation matrix may be an indication of violation of the assumption of underlying normality, but this is not necessarily so, because the nonpositive definiteness may also be from sampling variability.

It should be noted that the use of the tetrachoric correlation matrix for analysis based on the assumption of normal variables will usually provide vastly inflated chi-square values of fit and underestimated standard errors of estimates. This is because the tetrachoric correlations are considerably more variable than the regular Pearson product-moment correlations (also see the next section).

When feasible, the GLS estimator is to be preferred over the ULS estimator because of smaller sampling variation in the estimates and because it provides a chi-square test of model fit and standard errors of estimates. In actual practice, the use of the GLS estimator for dichotomous variables may present certain problems. Unlike the ULS estimator, it is a computationally very demanding estimator. It should probably not be attempted when the number of variables exceeds much more than 30 variables, and it already may be costly with fewer variables. To estimate the weight matrix properly with many variables, large samples are required—at the very least, 1,000 observations when more than 10 variables are present. With small samples, or with variables corresponding to rare events, the weight matrix may be poorly estimated and may become singular because of linear dependencies among the variable proportions. The latter problem may be avoided by deleting variables, although the choice of variables may introduce a certain amount of arbitrariness. LISCOMP prints the number of the variable for which the singularity occurs in order to guide in the search for variables to delete.

The GLS chi-square test suffers from the usual covariance structure-testing problem of proper choice of rejection region. The chi-square commonly appears to react unduly to small parameter changes with large samples and therefore provides a too sensitive instrument for judging model fit. A thorough analysis of rejection power may be very difficult if not impossible. Here, we offer only a very simple device for utilizing chi-square, taking sample size and number of restrictions (degrees of freedom) into account. Consider the chi-square value normed to a sample size of 1,000 (a large sample size in order to take the high tetrachoric variation into account) and divided by the number of restrictions. For example, a chi-square value of 300 for a sample size of 2,000 with a model having 100 degrees of freedom (100 restrictions) gives a value of $[300/(2000 \times 100)] \times 1000 = 1.5$. This index does not offer a probability statement for the testing of a hypothesis; such a probability statement is, in any case, most often distorted by complex sampling features and the use of models based on exploration of the data. The index should instead be viewed in a descriptive fashion and will be termed the model's descriptive fit value (DFV). When the model's DFV exceeds 1.5, the author's experience in factor-analysis contexts shows that either the model can be improved in substantively important ways or the variables are not suited to factor analysis, while a value much less than 1.0 suggest's an overfit, so that the model can be simplified.

As mentioned briefly above, the factor model and its estimation may also be expanded to the simultaneous analysis of several groups, investigating similarities and differences in the model across these groups. This feature will be utilized in the fourth section. When individuals of different groups respond to the same variables, it is of interest to study group invariance of measurement parameters, particularly the thresholds and loadings of (1), (2), and (3). This type of analysis was developed in Muthén and Christoffersson (1981). The estimation of the model is analogous to the least-squares approach just described, including GLS estimation with an appropriate weight matrix. In the multiple-group setting, the model also imposes restrictions on the threshold levels so that the analysis involves more than the tetrachoric correlations. For examples, see Muthén (1981). The analysis enables the comparison across groups of factor covariance matrices. Also, despite the fact that the analysis utilizes only dichotomous variables, the means of the continuous factors can be identified and estimated. This gives a more powerful way to compare levels among groups than the traditional use of sums of dichotomous items. The parameter identification and other technical issues for this model are complex and will not be described here; see Muthén and Christoffersson (1981) and Muthén (1987).

CONSIDERATIONS FOR MODEL USE

Although the use of tetrachoric correlations for factor analysis properly recognizes the categorical nature of the response variables, it has its own limitations. We have already mentioned the fact that tetrachorics are considerably more variable than regular Pearson product-moment correlations. Although this seems a disadvantage, the added variability correctly portrays the available information. The estimation of tetrachoric correlations is, however, difficult for variables representing rare events. Consider, for instance, two variables with a population distribution that is not uncommon for some of the Baltimore and Durham variables to be analyzed (for each site, there is a sample of approximately 3,500 observations): 1% of the population shows each of the two symptoms (corresponding to 40 individuals in one site), while only 0.2% shows both symptoms (seven individuals in one site). With these small probabilities in the cells of the bivariate table for the population, small frequency cells are likely to occur in samples and the influence of measurement error may be large. The tetrachoric correlation and its standard error may be estimated with considerable bias when expected cell frequencies fall below 5 (see Brown and Benedetti, 1977). A sample size of approximately 3,500 may seem large but for this reason is probably just large enough for describing relationships between such rare events. With a sample of this size and the cell probabilities just given, the tetrachoric correlation is 0.57 with a standard deviation of 0.09, and the correlation is therefore estimated with good precision. (We may note that the standard deviation for a correlation of this size between normal variables with a sample of 3,500 would only be 0.01. It is also interesting to note that the regular Pearson product-moment correlation coefficient, the phi coefficient, in this example is only 0.19.)

A crucial limitation is the assumption of normally distributed y^*'s. Continuing the example of a single depression factor, consider a symptom item that represents a very rare event in the population at hand. This means that the item has a strong positive skewness when scored 0/1 with 1 representing the presence of the symptom.

When studying a normal population, as we do here, this observed variable distribution can arise in at least two ways. We may have a normally distributed y^*, where the threshold is located at a point in the extreme right tail of the y^* distribution, reflecting the rareness of the event. A normal y^* variable is obtained when the factor and the error are both normal. Although normality seems a natural specification for a well-behaved residual, it is more arbitrary for the depression factor.

A non-normal factor and a normal residual would lead to a somewhat less normal y^*. Therefore, an alternative explanation for the positively skewed y is that the factor itself, and thereby

y^*, is positively skewed and the threshold more centrally located. In this second explanation, there is a large group of individuals with very similar low true scores on the depression factor, where a low score implies that they are unlikely to exhibit the symptom. In comparison, the first explanation states that considerably fewer individuals are similar in the low score range and this discrimination is because of a sizable number of individuals who are very unlikely to exhibit the symptom.

Either of the two alternative explanations may be suitable for different types of data. A relatively simple and plausible factor model that fulfills our assumptions of linear relations between the y^*'s and the factors in one application may hold true for non-normal factors, while in other cases normally distributed factors gives simplicity. In other words, using tetrachoric correlations would give biased results to the point of possibly not even indicating the correct number of factors.

The assessment of the appropriateness of the normality assumption for the y^*'s, with an implied assessment of factor normality, seems essential in dichotomous factor analysis. Nevertheless, it was not until recently that such an assessment was possible by the method of Muthén and Hofacker (1988). In that article, it is pointed out that the use of tetrachoric correlations implies the use of estimates from a model and as such should only be used if the model can be deemed to fit the data.

The model underlying the use of tetrachorics is one of underlying y^* normality. To test this assumption, one should in principle compare the predicted and observed cell frequencies in the table crossing all variables. However, even with huge sample sizes, this would yield many sparse cells, and usual test procedures would be grossly misleading for any realistic number of variables. On the other hand, the model cannot be tested on data from the marginal 2 × 2 table from which the correlation is usually computed, because in this case the model is "just-identified" and hence cannot be rejected. Muthén and Hofacker (1988) suggested the use of triplet testing where the benefit of large cell sizes is retained by considering trivariate marginal tables, while the model in each case is still overidentified and can be tested. For a given set of variables, a series of variable triplets can be tested for underlying trivariate normality with a one-degree-of-freedom chi-square test. If a small number of variables is involved in a large number of triplets showing rejection, those variables may contribute to violating the normality assumption. For variables not involved in rejections, the normality assumption may be retained and the corresponding tetrachorics used for further factor analysis. The procedure is included in the LISCOMP program.

If a large number of important variables are involved in rejection of normality, the question arises; What analysis can be used? A new alternative is discussed next.

ALTERNATIVE MODELING UNDER NON-NORMALITY

The use of tetrachoric correlations is incorrect when a non-normal factor model is found more plausible. Nevertheless, it would, of course, still be incorrect to revert to the use of phi coefficients. In some psychometric work, it is sometimes proposed to use sums of small sets of the dichotomous items. These sets will then have approximate interval-scale properties and could be used in a regular continuous-variable factor analysis. We may note that these new variables also are likely to be strongly skewed, although perhaps less so. Regular factor analysis on these variables may then better fulfill the assumptions of well-behaved residuals in linear relations. Although estimates are then not biased, any test of the number of factors may be distorted because of non-normal observed variables (for example, see Muthén and Kaplan, 1985). The choice of the items to form a set, however, is clearly ambiguous and should not be done unless preceded by an item-level factor analysis that determines which items belong to which factors—which brings us back to our original problem.

Let us consider an attempt to model under non-normal factors using an alternative approach proposed by Muthén (forthcoming). Assume that there is a large set of items that are hypothesized to measure a set of correlated factors. Assume further that it is relevant to view these "first-order" factors as a function of a single "second-order" factor. Using the latent response variable specification of the tetrachoric approach, we may then express the y^* variables ultimately as functions of the second-order factor through linear regressions. With a large number of items, the sum of all the items may be seen as a rough proxy for the second-order factor. In test item analysis, such a total sum is used to assess item measurement properties via "item-test" correlations. The distribution of the total sum reflects the assumed non-normality of the factors. Non-normality of the second-order factor induces non-normality of the first-order factors and non-normality of the y^*'s. The effect of the non-normal second-order factor on the y^*'s is dampened by the influence of normal residuals at each of the two levels. If an item is more strongly related to the total sum, its y^* is more strongly non-normal. The correlations among these non-normal y^*'s should then be used for factor analysis, revealing the structure of the first-order factors.

For simplicity, we may call the correlations among the non-normal y^*'s discussed above "non-normal tetrachorics." These correlations may, in fact, be estimated via the general model underlying LISCOMP as follows. Each y^* may be written as a linear regression function of the total sum of the items scored 0/1, say x,

$$y_j^* = \pi_j x + \delta_j \quad [6]$$

where π_j is a slope parameter and δ_j is a normally distributed residual with regular assumptions applied. Connecting y^* and y, as in (2), results in a nonlinear probit regression. For the vector of p y^* variables, we then have the covariance matrix

$$V(y^*) = \pi V(x)\pi' + \Omega \quad [7]$$

where π is the $p \times 1$ vector of slopes, V(x) is the variance of x, and Ω is the $p \times p$ covariance matrix of the residuals. The elements of π and Ω are parameters of a multivariate probit regression system and can be estimated by a two-stage maximum likelihood procedure in LISCOMP (for technical details, see Muthén, 1987). The Ω matrix is not diagonal. The diagonal elements of Ω are standardized to unity in these analyses. Inserting these values and the sample variance of x in

(7) gives us the estimated covariance matrix of the y^*'s. The corresponding correlations are the desired estimates of the non-normal tetrachoric correlations between the y^*'s. For each y^*, we can also calculate the estimated amount of y^* variance that is accounted for by x and thereby get a notion of the varying degree of non-normality in the y^*'s. Just as in test item analysis, the items for which we are estimating the probit coefficients may be excluded from the x variable in order not to obtain a spurious correlation. This effect, however, should be small with a large number of items.

FACTOR SCORE ESTIMATION

Once a well-fitting factor model has been found and its parameters estimated, it is of interest to estimate each individual's scores on the factors (see Mislevy, 1986). In this section, we will assume that a model with normally distributed factors is plausible. Factor score estimation is straightforward in ordinary factor analysis with continuous variables in that closed-form expressions exist for the calculation of the scores. In contrast, score estimation for dichotomous variables involves iterative optimization for each individual's score. Nevertheless, the calculations are not heavy. To calculate the scores—that is, the estimated value of the factor vector η_i for person i—we consider the distribution of η given person i's response vector y_i:

$$g(\eta \mid y) = \phi_\eta g(y \mid \eta) \,/\, g(y) \qquad [8]$$

This is the posterior distribution of y obtained by Bayes's Theorem, in which the normal density ϕ_η represents the prior distribution. Here, g(y) is the marginal distribution of y and $g(y \mid \eta)$ describes the measurement relations between η and y with components as in (4). With the usual assumption of conditional independence, this simplifies to the product of p terms such as in (4):

$$g(y \mid \eta) = P(y_1 \mid \eta)\, P(y_2 \mid \eta) \ldots P(y_p \mid \eta) \qquad [9]$$

Maximizing (8) with respect to the elements of η results in a Baye modal, or MAP (maximum a posterior), factor score estimator (se also Mislevy, 1986).

FACTOR ANALYSIS OF BALTIMORE DATA

The techniques described above will now be applied to the se of ECA data from Baltimore. There are 41 symptom variable: taken from the Diagnostic Interview Schedule, version III-A DSM-III, Robins and Helzer (1985). Measurements were ob tained for 3,835 individuals and an additional 86 individuals have missing data on all 41 items. In addition to these variables, we will also use eight variables created from the 41 items in order tc correspond to the eight groups of symptoms given as indications of a major depressive episode in the diagnostic criteria from the DSM-III. In our analyses, we will use data on 3,161 individuals who had no missing data on any of the 41 items. Inspection of the univariate statistics for the variables did not suggest that this reduction in sample size would lead to a biased sample.

ANALYSIS OF THE ORIGINAL 41 ITEMS

As a first analysis, all 41 items were subjected to an exploratory factor analysis using tetrachoric correlations. All exploratory analyses will use an oblique rotation by the promax method. For this set of 41 items, the ULS estimator was chosen because the GLS estimator was deemed to yield computations that were too unstable with this many strongly skewed variables. A model test of fit is then not available, and we will use the traditional scree plot of eigenvalues (for example, see Gorsuch, 1983: 166-169) from the tetrachoric correlation matrix as a rough guide to deciding the number of factors. The eigenvalues are plotted in Figure 1.

Figure 1 indicates two major factors, but it is difficult to see how many additional factors are needed. The two-, three-, four-, and five-factor solutions were inspected in terms of interpretability. The two-, three-, and four-factor solutions will be presented

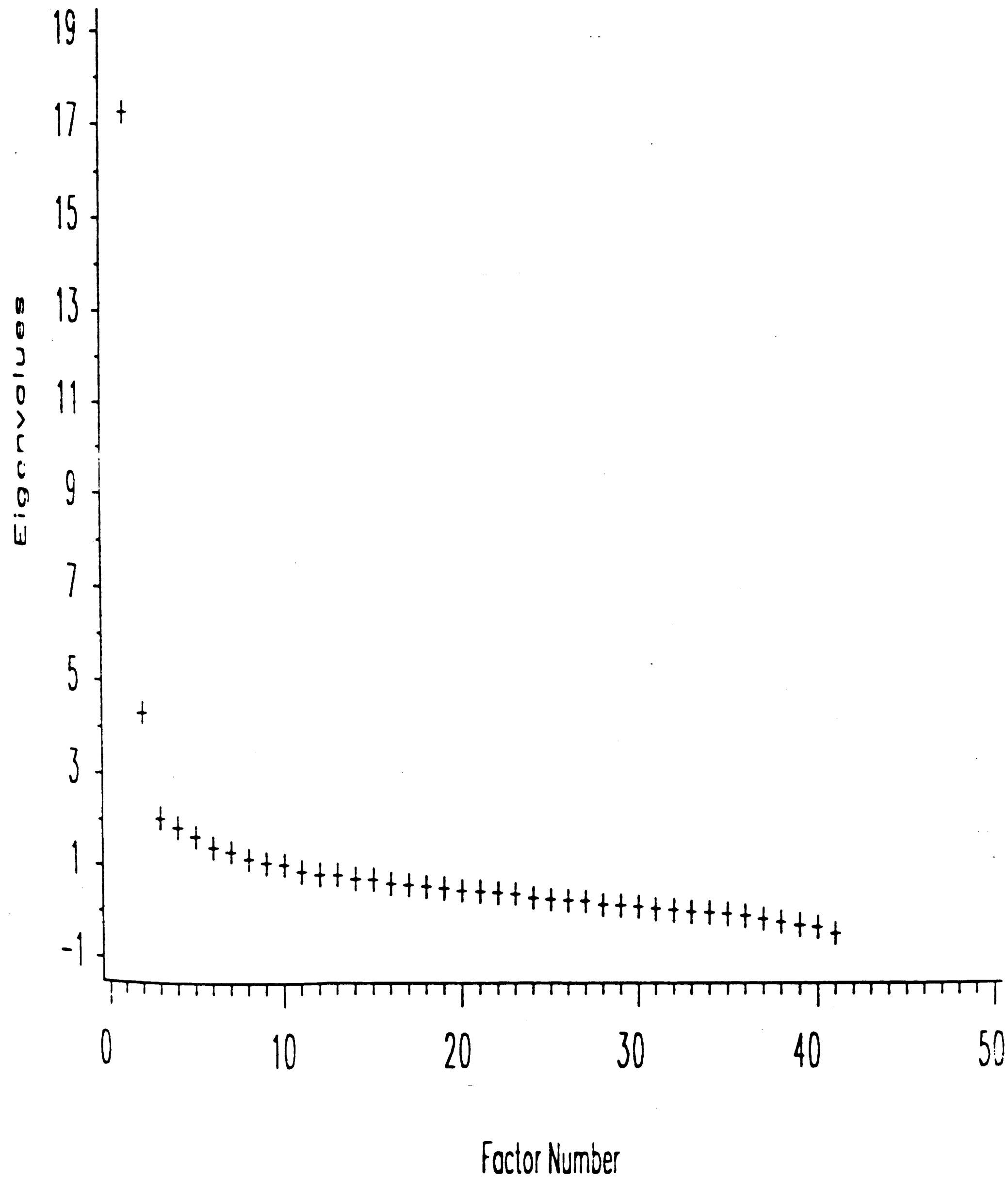

Figure 1: Scree Plot for Tetrachorics of 41 Items

here as they seemed the clearest. As we will see, the three- and four-factor solutions represent elaborations on one of the two factors in the two-factor solution.

The estimated factor loadings for the three solutions are given in Table 1. The factors are labeled Anxious Depression (AD) and

TABLE 1
Baltimore Data: ULS Factor Analysis of 41 Items

Variable	2-factor solution		3-factor solution			4-factor solution			
	AD	PA	AD	PA	SA	AD	PA	SA	S
Fear of being alone	0.36	0.44	0.53	0.42	-0.16	0.44	0.43	-0.22	0.17
Fear of animals	-0.06	0.71	0.08	0.69	-0.13	0.20	0.64	-0.05	-0.16
Short of breath	0.51	0.12	-0.04	0.10	0.76	-0.20	0.15	0.52	0.40
Fear of bugs	-0.12	0.79	0.03	0.76	-0.14	0.12	0.74	-0.11	-0.11
Fear of a closed place	0.03	0.73	-0.03	0.70	0.14	-0.09	0.74	0.02	0.14
Trouble thinking	0.79	0.01	0.74	-0.01	0.12	0.63	-0.05	0.09	0.24
Fear of a crowd	0.17	0.69	-0.08	0.67	0.37	-0.15	0.72	0.19	0.23
Crying spells	0.64	-0.01	0.50	-0.02	0.21	0.28	0.02	0.04	0.41
Thought about death	0.63	0.06	0.56	0.05	0.10	0.35	0.10	-0.07	0.42
Wanted to die	0.83	0.07	0.70	0.05	0.22	0.31	0.16	-0.12	0.70
Dizziness	0.50	0.10	0.02	0.08	0.66	0.08	-0.02	0.82	-0.04
Lost appetite	0.71	-0.06	0.84	-0.08	-0.13	0.73	-0.12	-0.09	0.19
Fear of eating in public	0.33	0.53	0.23	0.50	0.20	0.14	0.52	0.08	0.21
Fainting	0.44	-0.06	0.09	-0.07	0.50	0.17	-0.18	.0.69	-0.09
Panic attack	0.58	0.27	0.27	0.25	0.47	0.20	0.23	0.39	0.22
Eating increased	0.33	0.23	0.40	0.22	-0.05	0.53	0.11	0.11	-0.17
Fear of heights	-0.25	0.84	-0.13	0.81	-0.10	-0.00	0.79	-0.06	-0.17
Life hopeless	0.94	-0.19	0.68	-0.20	0.38	0.24	-0.08	0.01	0.79
Lost weight	0.49	0.08	0.58	0.07	-0.07	0.56	0.01	0.01	0.04
Moving all the time	0.51	0.23	0.58	0.22	-0.03	0.52	0.19	-0.05	0.14
Nervous person	0.52	0.14	0.31	0.12	0.32	0.24	0.11	0.25	0.20
Fear of going out alone	0.25	0.50	0.19	0.48	0.13	0.12	0.50	0.03	0.18
Heart beating hard	0.55	0.05	0.17	0.03	0.53	0.10	0.02	0.42	0.23

Dysphoria/anhedonia	*0.89*	*-0.09*	*0.84*	*-0.10*	*0.13*	0.66	-0.12	0.07	0.34
Sad for two years	*0.82*	*-0.12*	*0.54*	*-0.15*	0.40	0.37	-0.15	0.29	0.36
Interest in sex	*0.40*	*0.26*	*0.45*	0.24	-0.02	0.46	0.19	0.04	0.02
Trouble falling asleep	0.66	0.04	0.59	0.03	0.14	0.47	0.02	0.08	0.25
Sleeping too much	0.36	0.24	0.46	0.23	-0.08	0.63	0.10	0.12	-0.21
Fear of speaking to strangers	0.15	0.65	0.09	0.62	0.14	0.08	0.63	0.05	0.10
Fear of speaking in public	0.18	0.51	-0.07	0.50	0.38	-0.19	0.55	0.18	0.27
Fear of storms	0.00	0.64	0.08	0.61	-0.06	0.07	0.63	-0.10	0.04
Attempted suicide	0.91	-0.12	0.41	-0.15	0.71	-0.11	0.00	0.29	0.91
Thoughts of death	1.01	-0.24	0.85	-0.25	0.24	0.37	-0.13	-0.17	0.87
Thoughts slower	0.75	0.09	0.68	0.07	0.14	0.63	0.01	0.17	0.15
Tired	0.69	0.07	0.73	0.05	0.01	0.81	-0.06	0.12	-0.03
Talked more slowly	0.62	0.23	0.58	0.22	0.11	0.62	0.13	0.18	0.03
Fear of tunnels/bridges	-0.15	0.90	-0.11	0.87	0.01	-0.02	0.85	0.02	-0.09
Fear of water	-0.26	0.87	-0.18	0.87	0.01	-0.03	0.80	0.03	-0.21
Weakness	0.35	0.30	0.07	0.28	0.40	0.27	0.15	0.64	-0.25
Worthless, sinful, guilty	0.86	-0.07	0.86	-0.08	0.07	0.58	-0.04	-0.11	0.48

Factor Correlations

	2-factor solution		3-factor solution			4-factor solution			
	AD	PA	AD	PA	SA	AD	PA	SA	S
AD	1.00		1.00			1.00			
PA	0.57	1.00	0.52	1.00		0.52	1.00		
SA			0.53	0.38	1.00	0.45	0.39	1.00	
S						0.53	0.36	0.46	1.00

Phobic Anxiety (PA) in the two-factor solution because they seem to correspond to a traditional depression/anxiety division of items.[1]

The three- and four-factor solutions single out certain of the two-factor depression items to form two new factors. Both solutions show a Somatic Anxiety (SA) factor with items such as "Short of breath," "Dizziness," "Fainting," "Heart beating hard," and "Weakness." From a substantive point of view it is interesting to note that "Panic attack," representing "panic anxiety," and "Nervous person," representing "generalized anxiety," load on the SA factor. The four-factor solution also includes a Suicide (S) factor represented by the items: "Thought about death," "Wanted to die," "Life hopeless," "Attempted suicide," and "Thought of suicide." A difference between the three- and four-factor solutions is the "Crying spells" and "Life hopeless" load on the AD factor in the three-factor solution while on the S factor in the four-factor solution.

This first analysis of the 41 items used tetrachoric correlations based on the usual assumption of normality for the underlying latent response variables. Let us now carry out the analysis of these items by the alternative approach of non-normal tetrachorics described in the considerations for model use. The presumed non-normality is well reflected in the distribution of the sum of the 41 items scored 0/1. This sum has mean 1.45, variance 7.21, skewness 3.41, and kurtosis 15.18. Using the sum, LISCOMP's multivariate probit regression approach gave estimated y^* correlations that were then subjected to a ULS exploratory factor analysis. Note that, in principle, several subscores may be used instead of a single sum. A single sum was chosen here because a second-order factor conceptualization seems reasonable and because we do not want to prejudge the factor analysis by attempting to use subscores such as "Depression" and "Anxiety."

The non-normal tetrachorics are on the whole considerably lower than the normal tetrachorics. This happens despite the fact that the item-specific R^2's for the y^*'s are rather low, mostly ranging from 0.10 to 0.25 (see Table 2). The lower correlation values are presumably from the y^* skewness that is nevertheless induced by the strongly skewed sum. The differences across item

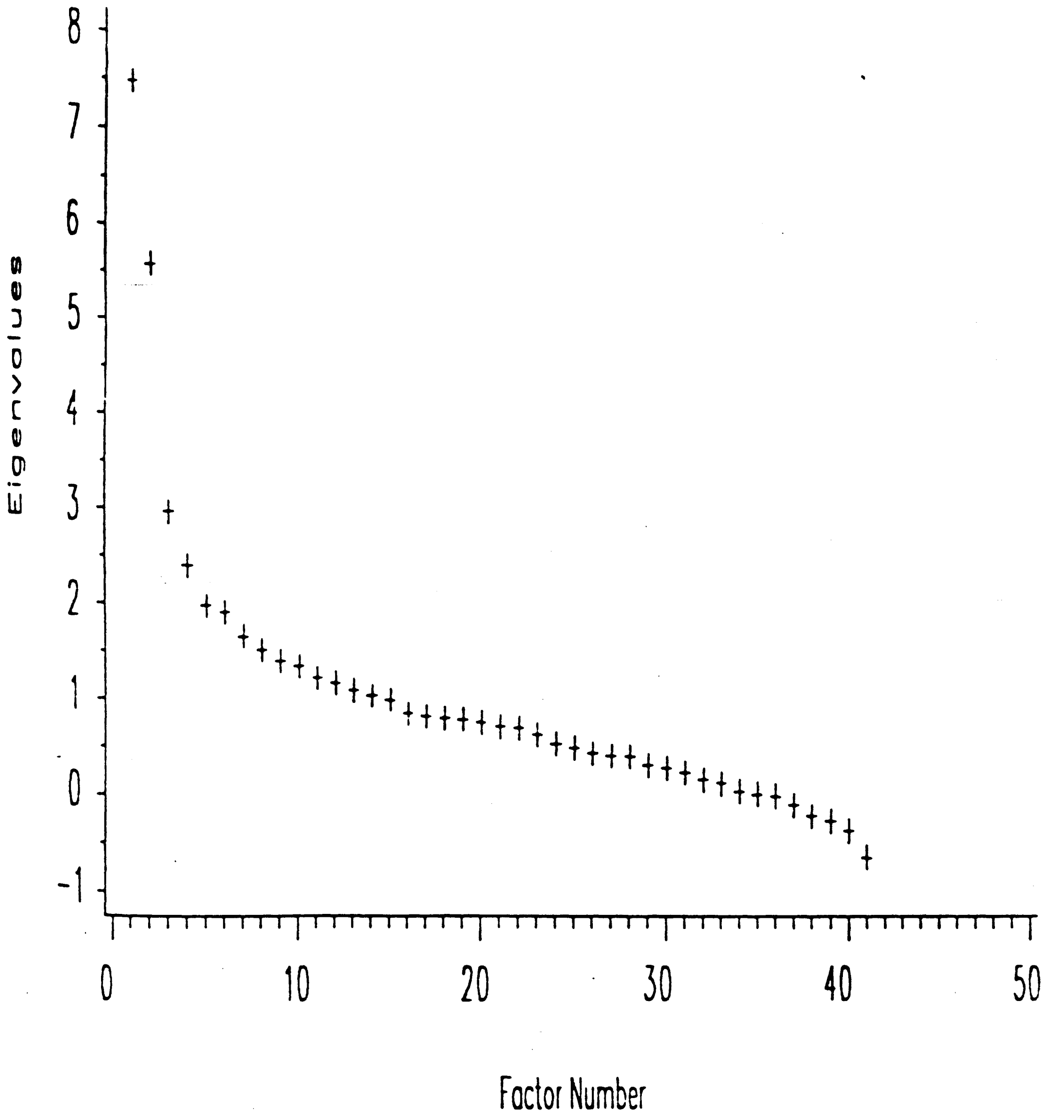

Figure 2: Scree Plot for Non-Normal Tetrachorics of 41 Items

R^2's gives an indication of the differential strength of relationship of each item with the total sum. We note that the items appear very homogeneous with the exception of "Fainting" and "Nervous person."

Figure 2 gives the scree plot for the non-normal tetrachorics of the 41 items. In comparison with Figure 1, we see a considerably

TABLE 2
Baltimore Data: ULS Factor Analysis of Non-Normal Tetrachorics for 41 Items

Variable	R^2	4-factor solution			
		AD	PA	SA	S
Fear of being alone	0.158	**0.437**	0.218	-0.109	-0.151
Fear of animals	0.116	0.193	**0.453**	-0.092	**-0.258**
Short of breath	0.103	-0.103	0.140	**0.521**	0.254
Fear of bugs	0.194	0.072	**0.629**	-0.058	-0.027
Fear of a closed place	0.162	-0.135	**0.617**	0.059	0.062
Trouble thinking	0.219	**0.596**	-0.145	0.057	0.106
Fear of a crowd	0.199	-0.106	**0.650**	0.194	0.135
Crying spells	0.159	**0.237**	0.063	0.112	**0.301**
Thought about death	0.215	**0.307**	0.075	-0.048	**0.320**
Wanted to die	0.250	0.208	-0.009	-0.268	**0.689**
Dizziness	0.112	0.040	0.051	**0.650**	0.057
Lost appetite	0.138	**0.486**	-0.019	-0.098	0.159
Fear of eating in public	0.153	0.138	**0.443**	-0.093	0.011
Fainting	0.046	0.249	-0.255	**0.792**	-0.307
Panic attack	0.196	0.093	0.189	**0.458**	0.130
Eating increased	0.109	**0.490**	0.026	0.081	**-0.265**
Fear of heights	0.151	-0.055	**0.644**	-0.013	-0.070
Life hopeless	0.220	0.122	-0.020	0.120	**0.840**
Lost weight	0.096	**0.447**	-0.026	0.063	-0.065
Moving all the time	0.187	**0.481**	0.094	0.054	-0.049
Nervous person	0.368	**0.192**	0.157	0.204	[illegible]

Fear of public transportation	0.197	-0.108	**0.790**	-0.001	-0.108
Dysphoria/anhedonia	0.246	**0.560**	-0.117	0.071	0.241
Sad for two years	0.144	**0.339**	-0.077	**0.239**	0.225
Interest in sex	0.132	**0.387**	0.145	-0.004	-0.012
Trouble falling asleep	0.247	**0.367**	0.005	0.118	0.154
Sleeping too much	0.125	**0.532**	0.099	0.034	-0.244
Fear of speaking to strangers	0.156	0.105	**0.557**	-0.121	-0.119
Fear of speaking in public	0.111	-0.104	**0.508**	0.013	0.174
Fear of storms	0.144	0.012	**0.514**	-0.075	0.061
Attempted suicide	0.121	-0.318	-0.076	0.053	**1.146**
Thoughts of death	0.163	0.255	-0.109	-0.161	**0.725**
Thoughts slower	0.230	**0.612**	-0.146	0.116	-0.014
Tired	0.256	**0.705**	-0.063	0.057	-0.072
Talked more slowly	0.232	**0.487**	-0.017	0.148	-0.052
Fear of tunnels/bridges	0.184	-0.147	**0.702**	0.076	-0.018
Fear of water	0.137	-0.042	**0.646**	0.027	-0.085
Weakness	0.118	0.254	0.097	**0.544**	-0.256
Worthless, sinful, guilty	0.214	**0.566**	-0.074	-0.148	**0.304**

Factor Correlations

	AD	PA	SA	S
AD	1.000			
PA	0.261	1.000		
SA	0.234	0.146	1.000	
S	0.354	-0.028	0.134	1.000

stronger indication of the need for more than two factors to represent the correlations.

The four-factor solution is remarkably similar to the four-factor solution for normal tetrachorics in Table 2 and yields exactly the same definition of the four factors. In contrast, the three-factor solution is quite different, lacking the clear distinction between the AD factor and the SA factor of Table 2. The four-factor solution is given in Table 2.

The factor correlations are much lower in Table 2 than in Table 1, reflecting the lower item correlations. Whereas the depression/anxiety factor correlation was about 0.5 with normal tetrachorics, it is only one-half of that with non-normal tetrachorics.

The partial convergence of factor interpretations for the normal and non-normal solutions is comforting. The factor analysis of latent response variable correlations shows a certain amount of robustness to alternative distributions for these variables. It is still of interest, however, to know what formulation fits the data best and we will return to this issue toward the end of our analysis of the Baltimore data in the section on the testing of normality assumptions.

ULS ANALYSIS OF 33 VARIABLES

In this section, we carry out an analysis in which items 74 through 89 (see Table 1 in Eaton and Bohrnstedt's introduction to this issue) are recoded to reflect whether a respondent is symptomatic on at least one of the items in each of the eight DIS/DSM-III groups of depression items. That is, eight new depression variables were created. The eight new variables were given the prefix "G" and created as follows. Appetite group (Group 1) = 1 if "Lost weight" = 1, "Eating increased" = 1, or "Lost appetite" = 1. Sleep group (Group 2) = 1 if "Trouble falling asleep" = 1 or "Sleeping too much" = 1. Slow/restless (Group 3) = 1 if "Talked more slowly" = 1 or "Moving all the time" = 1. Lost interest (Group 4) = 1 if "Interest in sex" = 1. Tired (Group 5) = 1 if "Tired out" = 1. Worthless (Group 6) = 1 if "Worthless, sinful, guilty" = 1. Trouble thinking (Group 7) = 1 if "Trouble concentrating" = 1 or "Thoughts slower" = 1. Thoughts of death (Group 8) = 1 if "Thought about death" = 1, "Wanted to die" = 1, "Thought of suicide" = 1, or "Attempted suicide" = 1. These eight variables will be factor analyzed together with the remaining 25 items, yielding a total of 33 variables for analysis. A separate analysis of the eight variables is presented in the section on the unidimensionality of depression and anxiety.

It is interesting to note that the combination of items for the eight grouped variables is in line with the factor analysis results for four factors in the section on the analysis of the original 41 items. In that section, however, we found that the items of the "Thoughts of death" variable did not load on the same factor as the items for the other grouped variables, but instead represented a separate dimension.

In the analysis of the 33 variables, we will use both the normal and non-normal tetrachoric approach. Figures 3 and 4 give the scree plot of eigenvalues for the two correlation matrices. Again, two to four factors will be examined.

Table 3 gives the exploratory factor solution with two, three, and four factors using normal tetrachorics. The solution for non-normal tetrachorics is here very similar for all solutions, except for the factor correlations. The four-factor solution is the preferred one because of its clear interpretability. Three factors are very similar to what was found for the 41 variable solution and will again be called Anxious Depression (AD), Phobic Anxiety (PA), and Somatic Anxiety (SA). Table 3 also shows a new factor having to do with public places and interaction with other people (PP for Public Phobia). The SA and PP factors both seem to be interesting adjoints to AD and PA in that they show factor correlations that are just about as high as between AD and PA. This is also true when using non-normal tetrachorics, although the factor correlations from the non-normal tetrachorics are again lower overall.

Note that for the four-factor solution, "Crying" and "Hopelessness" now load on AD. Note also that "Thoughts of death"—Group 8—now loads on the general depression factor, whereas in Table 1, its item components loaded on a separate factor. An

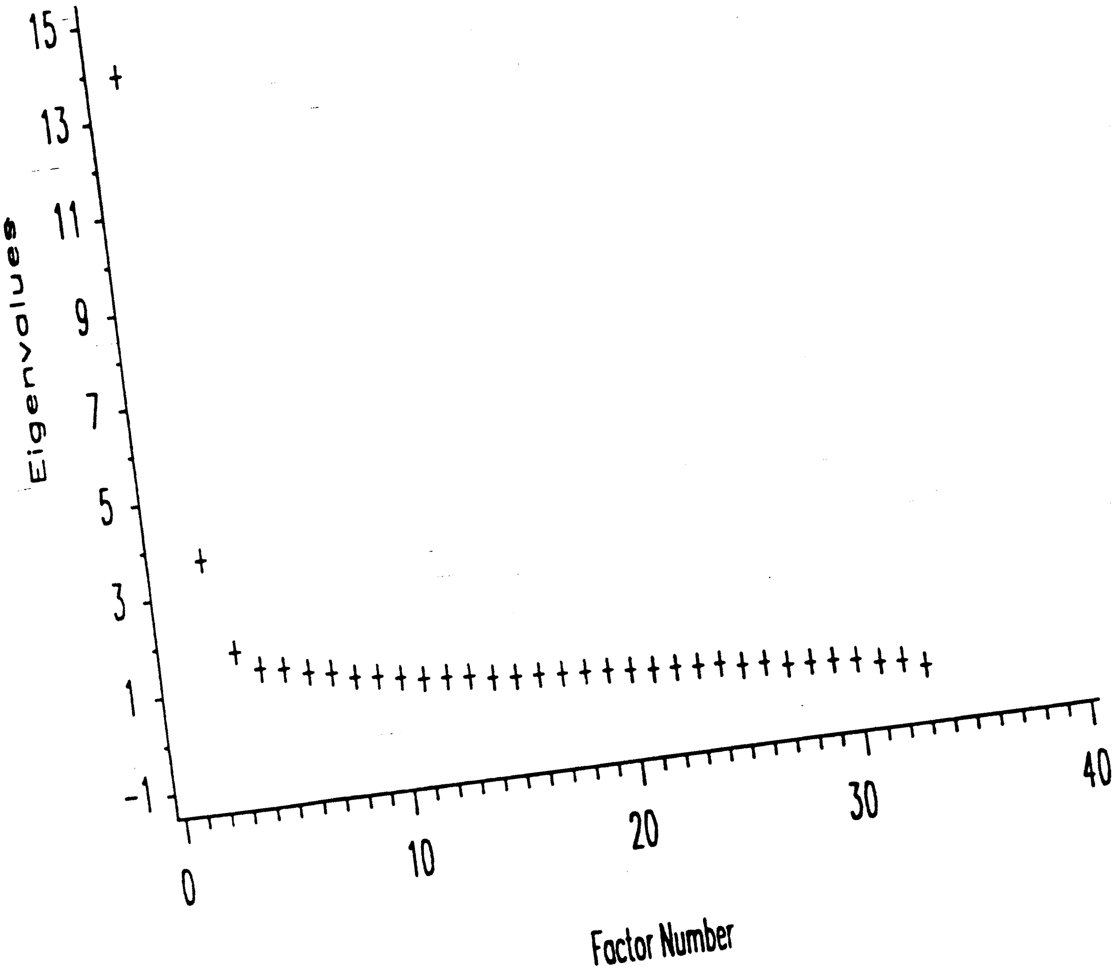

Figure 3: Scree Plot for Tetrachorics of 33 Items

explanation may be that "Thoughts of death"—Group 8 = 1—represents a considerably less extreme event than any of its item components, particularly the important items "Thought of suicide" and "Attempted suicide." Although the change in variable distribution per se should, in principle, not affect the tetrachoric solutions, the change in nature of the corresponding events may. It should be noted, however, that because in our analysis sample "Thought of suicide" = 1 and "Attempted suicide" = 1 correspond

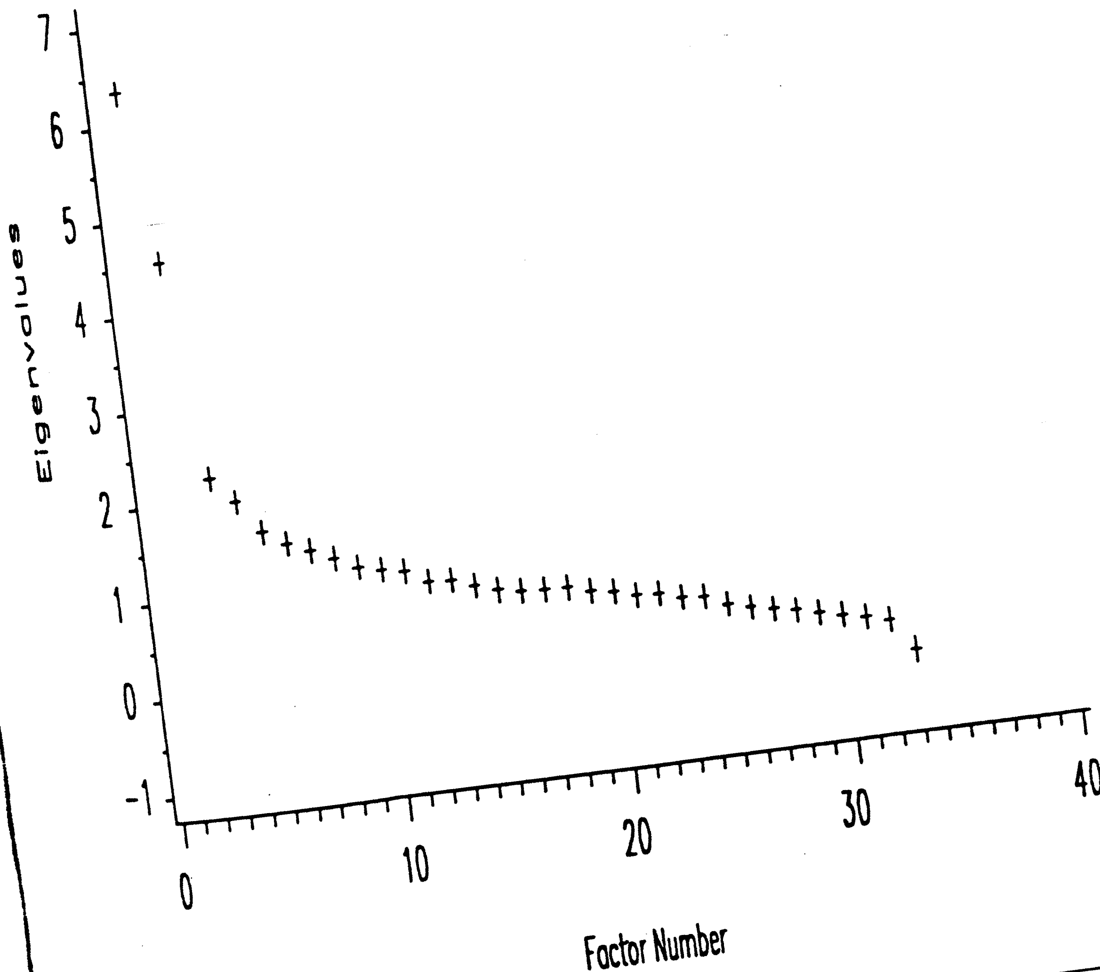

Figure 4: Scree Plot for Non-Normal Tetrachorics of 33 Items

to only 20 and 5 individuals, respectively, any correlations related to these items may be very poorly estimated. Given these considerations, a reanalysis was made of the 41 item set, excluding "Attempted suicide" and "Thought of suicide," and this made the suicide factor disappear while the "Thought about death" and "Wanted to die" items instead loaded on the AD factor.

In some applications, factor analysis is somewhat sensitive to the addition and deletion of items in the set analyzed. As seen

TABLE 3
Baltimore Data: ULS Factor Analysis of 33 Items

Variable	2-factor solution AD	2-factor solution PA	3-factor solution AD	3-factor solution PA	3-factor solution SA	4-factor solution AD	4-factor solution PA	4-factor solution SA	4-factor solution PP
Fear of being alone	**0.39**	**0.42**	**0.59**	**0.41**	-0.25	**0.60**	**0.43**	-0.19	-0.06
Fear of animals	0.02	**0.66**	0.16	**0.65**	-0.18	0.18	**0.63**	-0.13	0.01
Short of breath	**0.59**	0.07	0.22	0.07	**0.50**	0.21	-0.01	**0.44**	0.19
Fear of bugs	-0.06	**0.76**	0.03	**0.76**	-0.13	0.06	**0.71**	-0.10	0.07
Fear of a closed place	0.06	**0.72**	-0.01	**0.72**	0.09	0.01	**0.65**	0.10	0.13
Fear of a crowd	0.22	**0.66**	0.04	**0.66**	0.24	-0.02	**0.43**	0.14	**0.51**
Crying spells	**0.63**	-0.02	**0.64**	-0.03	0.02	**0.63**	-0.03	0.01	0.03
Dizziness	**0.57**	0.04	-0.08	0.03	**0.90**	-0.05	0.03	**0.86**	0.04
Fear of eating in public	**0.37**	**0.49**	**0.31**	**0.49**	0.09	0.24	0.25	-0.03	**0.52**
Fainting	**0.51**	-0.13	-0.00	-0.14	**0.71**	0.03	-0.05	**0.77**	-0.17
Panic attack	**0.64**	0.21	**0.33**	0.21	**0.43**	**0.31**	0.11	**0.37**	0.23
Appetite group - Group 1	**0.59**	0.10	**0.60**	0.09	0.02	**0.59**	0.09	0.03	0.01
Sleep group - Group 2	**0.69**	0.06	**0.66**	0.06	0.06	**0.68**	0.11	0.09	-0.10
Slow/restless - Group 3	**0.69**	0.19	**0.65**	0.18	0.08	**0.66**	0.22	0.11	-0.06
Lost interest - Group 4	**0.42**	**0.24**	**0.42**	0.23	0.01	**0.45**	**0.27**	0.05	-0.08
Tired - Group 5	**0.76**	-0.01	**0.74**	-0.02	0.06	**0.75**	0.02	0.07	-0.06
Worthless - Group 6	**0.83**	-0.07	**0.99**	-0.09	-0.15	**0.95**	-0.15	-0.20	0.15
Trouble thinking - Group 7	**0.75**	0.06	**0.70**	0.05	0.08	**0.71**	0.08	0.09	-0.04
Thoughts of death - Group 8	**0.62**	0.12	**0.69**	0.11	-0.07	**0.68**	0.09	-0.08	0.06
Fear of heights	-0.20	**0.81**	-0.20	**0.81**	-0.00	-0.16	**0.79**	0.05	0.01
Life hopeless	**0.88**	-0.16	**0.86**	-0.17	0.07	**0.83**	-0.18	0.05	0.06

Nervous person	0.57	0.08	0.39	0.08	0.25	0.39	0.07	0.24	0.05
Fear of going out alone	0.27	0.50	0.26	0.49	0.02	0.23	0.32	-0.03	0.29
Heart beating hard	0.61	-0.00	0.31	-0.00	0.41	0.27	-0.13	0.32	0.30
Fear of public transportation	-0.07	0.90	-0.10	0.90	0.04	-0.11	0.74	0.01	0.30
Dysphoria/anhedonia	0.89	-0.11	0.90	-0.12	0.01	0.91	-0.07	0.04	-0.10
Sad for two years	0.88	-0.21	0.73	-0.21	0.21	0.70	-0.27	0.14	0.18
Fear of speaking to strangers	0.22	0.60	0.20	0.60	0.03	0.09	0.27	-0.16	0.75
Fear of speaking in public	0.20	0.49	0.06	0.50	0.19	-0.05	0.21	0.06	0.64
Fear of storms	0.00	0.65	0.06	0.64	-0.08	0.11	0.66	-0.03	-0.06
Fear of tunnels/bridges	-0.12	0.89	-0.18	0.89	0.07	-0.13	0.91	0.15	-0.06
Fear of water	-0.16	0.81	-0.17	0.81	0.01	-0.15	0.74	0.03	0.11
Weakness	0.48	0.19	0.07	0.19	0.55	0.10	0.25	0.61	-0.11

Factor Correlations

	2-factor solution		3-factor solution			4-factor solution			
	AD	PA	AD	PA	SA	AD	PA	SA	PP
AD	1.00		1.00			1.00			
PA	0.55	1.00	0.54	1.00		0.47	1.00		
SA			0.59	0.39	1.00	0.56	0.30	1.00	
PP						0.51	0.45	0.43	1.00

above, this fact is here compounded by the existence of low-frequency items in which the corresponding correlations may be poorly estimated. With this in mind, the 33-variable factor analysis was inspected for effects of this kind. The item "Fainting" was found to be the lowest-frequency item with only 10 individuals reporting this symptom, whereas all other items had more than 30 individuals reporting. In contrast to other variables, most pairs of variables involving "Fainting" had less than five individuals reporting both symptoms, so the condition for good estimation of the corresponding item correlations is not fulfilled (see the section on considerations for model use). Deleting "Fainting" did not significantly alter the factor solutions, however.

GLS ANALYSIS OF DEPRESSION AND ANXIETY VARIABLES

As mentioned in the introduction, a major research question is the amount of overlap or separability between symptoms of anxiety and depression. From our previous analyses, it appears that one can clearly distinguish from among at least three factors: Anxious Depression, Phobic Anxiety, and Somatic Anxiety. The choice of factor solutions was made based on both scree plots and interpretability. It is of interest to attempt to statistically test the appropriateness of the suggested factors and study their separability. The testing will be accomplished by using the GLS estimator. Note that because the model has been arrived at from analysis of the data at hand, the test outcome should not be interpreted in a strict inferential sense, but merely as a descriptive measure of model fit. The separability of depression and anxiety is studied below, while the unidimensionality of each follows thereafter.

Separability of Depression and Anxiety

As an illustration of the approach to be considered, a subset of variables will be chosen to represent two of the factors, Anxious Depression and Phobic Anxiety. These are variables that appear to be particularly good indicators of either of the two factors while having small loadings on other factors. There are 21 variables that are candidates for this select pool of high quality indicators (they are listed in Table 4). Among them are the eight grouped variables and "Dysphoria/anhedonia" that measure Depression. Three more depression items appear promising: "Crying spells," "Life hopeless," and "Sad for two years." It is of interest to more closely study these 21 variables in a two-factor model for AD and PA.

An effective way to study the measurement properties for the 21 variables is as follows. An exploratory two-factor model is postulated. As proposed in Jöreskog (1969), we may carry out the exploratory model estimation in a correlation structure framework of a confirmatory factor analysis kind. Note that we are still using the normal tetrachoric correlations. We use a model with the same number of degrees of freedom and chi-square fit value as the exploratory two-factor model, but we perform the analysis by a correlation structure model that enables us to readily obtain standard errors of the estimated loadings.

The exploratory two-factor model needs to impose four restrictions on the loadings and factor covariance matrix elements in order to eliminate the indeterminacies of this model. Effectively, the exploratory solution does this automatically by the promax rotation. In the correlation structure framework, we may instead impose these restrictions by standardizing the factor variables to one as in the exploratory solution, allowing the factors to correlate, and choosing two variables to represent each of the two factors by forcing each of the variables to represent each of the two factors by forcing each to have a zero loading on the other factor. In the correlation structure approach, we also make use of the possibility to include modification indices for the off-diagonal residual correlations. Although these are fixed to zero, large index values for certain items indicate that the items in question may not fit the simple structure as well as the others.

In the correlation structure estimation of the exploratory two-factor model for the 21 variables, "Fear of bugs" was chosen to represent the anxiety factor and "Tired"—Group 3—the depression factor. The chi-square test gave a value of 384 with 169 degrees of freedom, corresponding to a DFV of 0.7. The DFV for the exploratory one-factor model was 1.7 and the DFV for three

TABLE 4
Baltimore Data: Parameter Estimates and Z-values for 21 Items

Variable	AD		PA	
Fear of animals	0.009	(0.192)	0.825	(18.420)
Fear of bugs	0.000	(0.000)	0.797	(43.577)
Fear of a closed place	0.130	(2.791)	0.765	(20.067)
Crying spells	0.733	(16.828)	-0.169	(-3.020)
Appetite group - Group 1	0.653	(15.285)	0.080	(1.410)
Sleep group - Group 2	0.727	(20.074)	0.095	(1.964)
Slow/restless - Group 3	0.743	(20.862)	0.228	(4.988)
Lost interest - Group 4	0.474	(8.686)	0.341	(5.123)
Tired - Group 5	0.810	(44.938)	0.000	(0.000)
Worthless - Group 6	1.021	(22.583)	-0.259	(-3.948)

Trouble thinking - Group 7	0.880	(22.964)	-0.054	(-1.025)
Thoughts of death - Group 8	0.782	(20.580)	-0.042	(-0.800)
Fear of heights	-0.123	(-2.387)	0.836	(22.549)
Life hopeless	1.161	(24.704)	-0.485	(-7.025)
Fear of public transportation	-0.062	(-1.184)	0.908	(25.269)
Dysphoria/anhedonia	0.955	(25.037)	-0.157	(-2.594)
Sad for two years	1.013	(20.978)	-0.387	(-5.583)
Fear of speaking to strangers	0.341	(7.901)	0.564	(11.681)
Fear of storms	0.151	(2.906)	0.636	(15.141)
Fear of tunnels/bridges	0.036	(0.675)	0.930	(25.702)
Fear of water	-0.007	(-0.139)	0.799	(19.797)

factors was 0.5. This indicates that while one factor is not sufficient, the two-factor solution can be further simplified—for instance, by imposing a simple structure where many items are allowed to load on only one of the two factors. This will not be attempted here, however, because of lack of space. Inspecting the modification indices for the residual correlations did not reveal patterns of unduly high values for any items. The estimates and estimate by standard error ratios are given in Table 4. The ratios are approximately normally distributed. Because the sample has not been obtained by simple random sampling, significance assessment should be done in very rough terms. The estimated factor correlation is 0.56 with estimated standard error 0.04.

Table 4 gives interesting information about the symptom overlap in the measurement of Anxious Depression and Phobic Anxiety. Consider the item "Lost interest"—Group 4—"Was there ever a period of several weeks when your interest in sex was a lot less than usual?" It is included as a diagnostic criterion for depression in the DSM-III but has an equally high and significant loading on the PA factor. This item is therefore not useful for discriminating between AD and PA. Among the three depression items that were not included in the DSM-III set of eight, "Dysphoria/anhedonia" and "Crying spells" would seem to be a good additional diagnostic criterion, while "Life hopeless" and "Sad for two years" are not. Among anxiety items, "Fear of speaking to strangers" does not perform well. Deleting poor items for the Baltimore data leaves us with nine excellent depression indicators—"Crying spells"; appetite group (Group 1); sleep group (Group 2); slow/restless (Group 3); tired (Group 5); worthless (Group 6); trouble thinking (Group 7); thoughts of death (Group 8); "Dysphoria/anhedonia"; and eight excellent anxiety indicators—"Fear of animals," "Fear of bugs," "Fear of a closed place," "Fear of heights," "Fear of public transportation," "Fear of storms," "Fear of tunnels/bridges," and "Fear of water."

Unidimensionality of Depression and Anxiety

It is of interest also to consider whether the variables used in the DSM-III diagnosis of depression follow a one-factor model. There are nine such variables—appetite group (Group 1); sleep group (Group 2); slow/restless (Group 3); lost interest (Group 4); tired (Group 5); worthless (Group 6); trouble thinking (Group 7); thoughts of death (Group 8); and "Dysphoria/anhedonia." A GLS analysis resulted in an excellent fit to the one-factor model with a chi-square of 43 with 27 degrees of freedom and a DFV of 0.5. This analysis strongly supports the notion of a unidimensional trait underlying the items used in the DSM-III diagnosis of depression. The estimated factor loadings show a high degree of item homogeneity. Using the item order above, they are: 0.70, 0.73, 0.83, 0.86, 0.64, 0.85, 0.85, 0.71, and 0.84. The nine new AD items also fit a one-factor model well with a chi-square of 50 with 27 degrees of freedom and a DFV of 0.6.

The one-factor analyses, however, do not address the question of separability of anxiety and depression. For instance, the two-factor analysis of the preceeding section reveals that the item "Lost interest"—Group 4—does not discriminate well between AD and PA. The fact that "Lost interest"—Group 4—partly measures a factor other than AD could not be detected in the present one-factor analysis; a misfitting one-factor model would only occur if more than one item measured such another factor.

TESTING OF NORMALITY ASSUMPTIONS

The nine depression variables and the eight anxiety variables may be used to judge whether underlying normality is plausible for these types of symptoms. This will be assessed by the triplet testing method of underlying y^* normality discussed in the section on methods. Because each set of variables represents its own factor, the triplet testing will be carried out on each set of variables separately. Triplet testing on all 41 items would have been computationally too demanding, and therefore the earlier finding of convergence of the normal and non-normal tetrachoric factor solutions is valuable regardless of the outcome of the present testing.

For the nine depression variables, there are 81 triplets to be tested. It turned out that only one triplet rejected normality at the 1% level—"Slow/restless" (Group 3); "Worthless" (Group 6); and "Trouble thinking" (Group 7)—while two triplets gave rejection at the 5% level—appetite group (Group 1); sleep group (Group 2); tired (Group 5); and tired (Group 5); trouble thinking (Group 7); and thoughts of death (Group 8). Given the number of triplets tested, this does not indicate that normal tetrachorics would be inappropriate. For the eight anxiety variables, none of the 56 triplets gave rise to a normality rejection. This gives remarkably strong support for choosing the description of the depression and anxiety factors as normally distributed (discussed in the section on methods).

CLASSIFICATION OF INDIVIDUALS

It is of interest to consider the classification of individuals using the new set of excellent AD indicators as compared to the depression classification obtained by DSM-III. In both cases, nine items are used. In both cases, the eight items—appetite group (Group 1); sleep group (Group 2); slow/restless (Group 3); tired (Group 5); worthless (Group 6); trouble thinking (Group 7); thoughts of death (Group 8); and "Dysphoria/anhedonia"—are used, while the new set adds "Crying spells," and the DSM-III set adds Lost interest (Group 4).

In the DSM-III classification, a person is diagnosed as suffering from a major depressive disorder if the person admits to the dysphoria/anhedonia symptom and has at least four of the other eight symptoms. On the other hand, a logical use of the factor analysis model that we have established would lead to judging a person by the scale value on the AD factor. This then leads to the use of estimated factor scores to determine the level of AD where a certain cutoff point for major depressive disorder may be relevant. As is the case of using a simple sum of the nine symptoms, the weighting of the items is, however, a (nonlinear) function of both the loading sizes and the prevalence of the symptom—that is, the item's threshold value (see equation (3)).

Although the item set is very similar, the DSM-III classification that is partly conditional and partly summed-score based is different from the factor analysis classification that utilizes estimated measurement parameters to form factor scores that draw on nonlinearly weighted summed scores. The cutoff point for a depression classification based on the AD factor scale is not an obvious one.

The quality of the two classification schemes must be judged by variables external to the present data set, and classification by the factor analysis model does not automatically correspond to clinically relevant categories. Still, it is of interest to study the agreement of the two schemes. By the DSM-III classification, our analysis sample of 3,161 contains 48 individuals diagnosed as suffering from a major depressive disorder. We may calculate the estimated factor scores for all 3,161 individuals in our sample and note the location of estimated values for these 48 individuals. We find 52 undiagnosed individuals with estimated factor scores at least as large (AD value at least as high) as the lowest value for these 48, although none having a higher value than the highest. In this way the use of the factor model could approximately double the number of diagnosed individuals. Table 5 shows stem-and-leaf diagrams of the factor score distributions for the two sets of 48 and 52 individuals. It is seen that the majority of the 52 undiagnosed individuals are in the lower range of the scores of the 48 diagnosed ones. There are 10 individuals among the 52 who have values at least as high as the median score of 2.04 for the 48. These 10 all have at least 5 symptoms, but none has the symptom of dysphoria/anhedonia.

The use of estimated factor scores can also be expanded to several factors. We may, for instance, estimate bivariate scores for AD and PA using both the set of nine new depression items and the set of eight anxiety items. Because of the high factor correlation of 0.56, the use of the PA items contributes substantially to the precision of the estimation of AD scores. Standard errors of estimates can also be computed.

TABLE 5
Baltimore Data: Factor Score Distribution of 48 DSM-III Depression-Classed Individuals Compared to 52 Unclassified Individuals

Stem-and-leaf*

Not classified as depressed (N=52)		Classified as depressed (N=48)	
		29	22
		28	
		27	
		26	147
		25	45
24	9	24	4
23	3355	23	04446
22	0	22	226788
21	18	21	9
20	003348	20	134447
19	0233455559	19	26777899
18	000233444889	18	3
17	5556777899999999	17	022333479
		16	9
		15	
		14	226

*An entry such as 21 18 should be understood as two observations, both with stem value 21 and with leaf values of 1 and 8, respectively. In the scale used, this means observation values of 2.11 and 2.18, respectively.

COMPARATIVE ANALYSES ON DURHAM DATA

The analysis results using Baltimore data will now be briefly compared to those using data from Durham. The analysis sample consists of 3,542 individuals. We will not repeat all analysis steps carried out for Baltimore data but concentrate on the 33 variable analysis by ULS and the analysis of the 21 selected variables by GLS. Simultaneous analysis of the two sites will also be performed.

In the exploratory analyses of the 33 variables, the two-, three-, and four-factor solutions were inspected. The scree plot of eigenvalues was similar to that of Baltimore, shown in Figure 3. The two-factor solution was very close to that of the Baltimore analysis. The three-factor solution resulted in the PA factor, while the two other factors did not correspond to the AD and SA factors as defined earlier. The four-factor solution had factor definitions similar to the Baltimore analysis. The three- and four-factor solutions are given in Table 6. Major deviations from the Baltimore four-factor solution are that "lost interest" (Group 4) and "worthless" (Group 6) load weakly on AD. An investigation of the sensitivity of the factor solutions to low-frequency items was also done for the Durham data. In contrast to other variables, "Fear of animals" was involved in many pairs of variables in which less than five individuals reported both symptoms. Recalculating the four-factor solution without the "Fear of animals" item did not alter the solution significantly.

Because the 33 variable four-factor solution deviated little from the corresponding Baltimore analysis, it was decided to submit the same subset of 21 variables as in the section on GLS analysis of depression and anxiety variables for further analysis. As for Baltimore, the exploratory GLS analysis of one to five factors indicated that the two-factor solution was adequate. The descriptive fit values for one to three factors were 2.6, 1.1, and 0.7, with a two-factor chi-square of 629 with 169 degrees of freedom.

The exploratory two-factor analysis was again repeated in a correlation structure framework with estimation of standard errors

TABLE 6
Durham Data: ULS Factor Analysis of 33 Items

Variable	3-factor solution			4-factor solution			
	I	PA	III	AD	PA	SA	PP
Fear of being alone	0.20	0.21	**0.52**	0.06	0.22	0.13	**0.54**
Fear of animals	0.08	**0.55**	-0.00	0.23	**0.53**	-0.18	0.05
Short of breath	**0.68**	-0.02	-0.09	0.20	0.01	**0.69**	-0.21
Fear of bugs	-0.06	**0.61**	0.20	0.05	**0.60**	-0.14	0.24
Fear of a closed place	-0.03	**0.51**	**0.41**	-0.18	**0.52**	0.18	**0.39**
Fear of a crowd	0.13	**0.35**	**0.54**	-0.12	**0.37**	**0.26**	**0.52**
Crying spells	**0.50**	-0.04	0.23	**0.36**	-0.05	0.12	**0.27**
Dizziness	**0.61**	0.05	0.02	0.14	0.08	**0.65**	-0.07
Fear of eating in public	0.20	0.07	**0.61**	0.24	0.05	-0.18	**0.74**
Fainting	**0.44**	-0.24	0.24	0.18	-0.24	**0.28**	**0.25**
Panic attack	**0.49**	0.10	**0.30**	0.25	0.10	**0.27**	**0.30**
Appetite group - Group 1	**0.56**	0.04	0.06	**0.48**	0.03	0.06	0.11
Sleep group - Group 2	**0.60**	-0.05	0.16	**0.39**	-0.05	**0.23**	0.19
Slow/restless - Group 3	**0.67**	-0.02	0.15	**0.51**	-0.02	0.16	0.20
Lost interest - Group 4	**0.39**	**0.23**	0.03	**0.20**	**0.24**	**0.26**	-0.00
Tired - Group 5	**0.72**	-0.08	0.08	**0.56**	-0.09	0.16	0.13
Worthless - Group 6	**0.40**	-0.17	**0.55**	0.23	-0.16	0.11	**0.60**
Trouble thinking - Group 7	**0.75**	0.13	-0.04	**0.71**	0.11	0.03	0.03
Thoughts of death - Group 8	**0.72**	-0.02	0.02	**0.70**	-0.05	-0.01	0.10
Fear of heights	0.20	**0.77**	-0.17	0.15	**0.76**	0.14	-0.20
Life hopeless	**0.78**	-0.05	0.03	**0.77**	-0.08	-0.03	0.13

Nervous person	**0.46**	-0.02	0.22	**0.28**	-0.01	0.19	**0.24**
Fear of going out alone	0.15	0.32	**0.44**	0.14	**0.31**	-0.05	**0.49**
Heart beating hard	**0.53**	0.07	0.01	-0.07	0.12	**0.87**	-0.16
Fear of public transportation	-0.04	**0.83**	0.00	0.06	**0.81**	-0.08	0.01
Dysphoria/anhedonia	**0.93**	0.13	-0.33	**1.01**	0.10	-0.04	-0.27
Sad for two years	**0.88**	0.16	-0.21	**0.95**	0.13	-0.06	-0.14
Fear of speaking to strangers	-0.20	0.28	**0.80**	-0.14	0.27	-0.21	**0.88**
Fear of speaking in public	0.09	**0.48**	0.24	0.08	**0.47**	0.01	**0.25**
Fear of storms	0.08	**0.58**	0.11	-0.02	**0.59**	0.16	0.07
Fear of tunnels/bridges	-0.03	**0.77**	0.05	-0.12	**0.78**	0.17	-0.00
Fear of water	-0.05	**0.73**	0.06	0.02	**0.72**	-0.05	0.05
Weakness	**0.54**	-0.10	0.20	0.01	-0.07	**0.67**	0.12

Factor Correlations

	3-factor solution				4-factor solution			
	I	PA	III		AD	PA	SA	PP
I	1.00			AD	1.00			
PA	0.36	1.00		PA	0.32	1.00		
III	0.59	0.41	1.00	SA	0.66	0.27	1.00	
				PP	0.61	0.41	0.59	1.00

just as in the corresponding Baltimore analysis. The estimates and estimate/standard error ratios are given in Table 7.

Just as for the Baltimore data, the "lost interest" (Group 4) variable appears to be a poor indicator of depression; in fact, it is here a better indicator of anxiety. As for Baltimore, "Life hopeless" and "Sad for two years" also emerge as poor indicators of depression in terms of also being strongly influenced by anxiety; but, in addition, "Crying spells," appetite group (Group 1), and "Dysphoria/anhedonia" discriminate rather poorly between depression and anxiety. As for Baltimore, "Fear of speaking to strangers" is not a good indicator of anxiety.

It is of interest to study more carefully the similarities and differences in the factor solutions for the two sites. This can be effectively carried out in a simultaneous analysis of the data for the two sites, using the approach briefly mentioned in the section on methods. The idea is to do a single-factor analysis of both sets with equality constraints across groups imposed on the measurement parameters for each of the variables. Because the same measurement instrument is used, this is a natural test of measurement invariance. If invariance can be established, it makes sense to go further and investigate the similarity of the factor mean vector and covariance matrix.

In our previous analyses of the 21 variables, four variables were found to give poor discrimination between AD and PA for both sites. As in the normality testing and classification studies of the sections on the testing of normality assumptions and the classification of individualism, we will therefore delete these four variables and study factor invariance by means of nine AD items and eight PA items, a total of 17 variables. These are also the nine and eight items deemed as excellent indicators in the Baltimore analysis. The simultaneous analysis of the two sites involves estimation with a weight matrix that not only contains weights for tetrachoric correlations as before but also weights for thresholds. An analysis of the weight matrices showed that near-singularity in this weight matrix occurred for the Durham data because of strong covariation between thresholds and correlations. Inspection revealed that this was caused by a single item, "Fear of

TABLE 7
Durham Data: Parameter Estimates and Z-values for 21 Items

Variable	AD		PA	
Fear of animals	-0.209	(-4.792)	0.786	(21.566)
Fear of bugs	0.000	(0.000)	0.694	(30.723)
Fear of a closed place	0.132	(3.247)	0.707	(37.365)
Crying spells	0.541	(16.382)	0.319	(8.911)
Appetite group - Group 1	0.601	(18.004)	0.375	(12.225)
Sleep group - Group 2	0.652	(28.784)	0.073	(2.547)
Slow/restless - Group 3	0.788	(39.323)	0.101	(3.512)
Lost interest - Group 4	0.368	(7.048)	0.693	(19.619)
Tired - Group 5	0.839	(48.314)	0.000	(0.000)
Worthless - Group 6	0.829	(30.287)	0.186	(5.688)
Trouble thinking - Group 7	0.798	(35.983)	0.198	(6.601)
Thoughts of death - Group 8	0.756	(38.982)	0.137	(5.000)
Fear of heights	0.047	(1.042)	0.808	(38.410)
Life hopeless	0.769	(33.703)	0.268	(8.577)
Fear of public transportation	-0.117	(-2.419)	0.849	(44.442)
Dysphoria/anhedonia	0.793	(32.986)	0.328	(11.277)
Sad for two years	0.899	(25.808)	0.483	(14.131)
Fear of speaking to strangers	0.440	(8.762)	0.858	(26.000)
Fear of storms	0.058	(1.472)	0.616	(23.911)
Fear of tunnels/bridges	0.020	(0.410)	0.898	(40.868)
Fear of water	-0.045	(-1.160)	0.575	(22.541)

animals." This item was therefore deleted in both data sets and the analysis carried out on the remaining 16 items.

Our first model takes the thresholds and the loadings to be equal across the two groups of individuals. The loading pattern is taken from Tables 4 and 7, and the factors are allowed to correlate. The factor mean vectors and covariance matrices are allowed to vary across groups, representing possible differences in factor level and dispersion. The chi-square value of fit is 369 with 204 degrees of freedom and a DFV of 0.3. This is an excellent fit, although we must be aware that it is partly from the trimming of the model by deletion of less than perfect indicators. A better way to look at fit when comparing two groups is to consider the worsening of fit induced by the equality constraints on the thresholds and the loadings. Without the equality constraints, the chi-square is 289 with 178 degrees of freedom, and a DFV of 0.2. The interesting statistic is the difference in fit values, where the chi-square difference is obtained as 80 with 26 degrees of freedom and a DFV of 0.5. Hence, the measurement invariance does not seem to be rejected by data, and the model fits quite well. It may be noted that the equality of loading pattern may be hard to detect in Tables 4 and 7 because in these tables differences in factor variances across groups are not separated out. In the simultaneous analysis, the factor covariance matrix is allowed to vary across groups. The measurement invariance across the two sites is reassuring. This means, for instance, that it would be correct to use the same depression-classification instruments based on the nine new AD items in both sites and the classification would be comparable.

Given measurement invariance, the next logical step is to test for invariance of the factor covariance matrices across the sites. This gives a chi-square difference value of 16 with 3 degrees of freedom and a DFV of 0.8. Hence, there is no strong indication of differences in the factor variances or in the correlation across sites. However, imposing the further restriction of equal means for the AD and PA factors, respectively, gave a considerably stronger indication of site difference with a chi-square difference value of 26 with 2 degrees of freedom and a DFV of 2.0.

We conclude that there seems to be strong evidence of measurement invariance, little evidence of factor variance or correlation difference, and strong evidence of factor mean differences across the two sites. The common factor correlation is estimated as 0.4, and the factor mean differences are estimated as 0.15 (standard error 0.05) and 0.19 (standard error 0.05), indicating lower AD and PA levels in Durham than in Baltimore. Related in a more directly interpretable way, Durham's AD factor mean is about 18% of a (AD factor) standard deviation lower than Baltimore's, while the PA factor mean is about 27% of a (PA factor) standard deviation lower.

SUMMARY

This article has examined the special methodological issues related to the factor analysis of symptom variables and has applied appropriate techniques to several sets of ECA data on depression and anxiety. Following are summaries of methodological issues and substantive findings.

METHODOLOGICAL ISSUES

The general problem was how to properly specify a model for analyzing the dimensions underlying a set of strongly skewed dichotomous items and how to best use a model for the classification of subjects. The article discussed one model that is appropriate when one wants to conceptualize the factors as continuously distributed. Two model parts were discussed, the specification of the relations between the symptoms and the factors and the specification of the factor distributions.

The chosen specification assumed that the items were related to a set of factors through a nonlinear model, thereby avoiding the problems of dichotomous variables encountered when using the linear factor analysis model. It was pointed out that the same nonlinear specification has been found useful in the context of logit and probit regression in bioassay and Item Response Theory for educational tests. The specification was described as a linear model for continuous latent response variables, y^*'s, underlying each item.

The factors were specified as normally distributed and leading to normally distributed latent response variables and the use of tetrachoric correlations. It was pointed out that this was a more crucial part of the model specification. Techniques were described for testing the normality assumptions on the latent response variables and for allowing non-normal latent response variables.

Unweighted and weighted least-squares factor analysis were described for tetrachoric correlations. Model tests of fit and the analysis of one and several groups of individuals were reviewed. The specific problem of a small number of individuals reporting both of two symptoms was discussed. This problem makes it necessary to have very large sample sizes to avoid problems in computing correlations and weights for use in the weighted least-squares technique.

Factor score estimation was proposed to obtain each individual's values on the factors as well as to enable a classification of the individuals.

SUBSTANTIVE FINDINGS

Using the preferred methodology described above, a series of complementary analyses were carried out for the Baltimore and Durham data sets. Extensive analyses were carried out for Baltimore. The Durham data was used for a more limited set of analyses for comparison with the Baltimore findings. The Baltimore results will be summarized first.

The nine variables used for the DSM-III diagnosis of depression (see the section on the unidimensionality of depression and anxiety) were found to represent a single dimension. All items measured this factor well. In analyses involving the items that were used to create the nine variables, the unidimensionality was also supported (see the sections on the analyses of variables).

The complete set of DIS items resulted in a four-factor structure in which the depression factor above was found to be of major importance (ULS analysis of 33 variables). These factors were termed (see Table 3): Anxious Depression (key items: the nine DSM-III items, "Life hopeless," "Crying spells"), Phobic Anxiety (key items: "Fear of tunnels/bridges," "heights," "water," "public transportation," "bugs," "storms," "animals," "closed place"), Somatic Anxiety (key items: "Dizziness," "Fainting," "Weakness"), Public Places (key items: "Fear of speaking to strangers," "Fear of speaking in public," "Fear of eating in public," "Fear of a crowd").

Separability of depression and anxiety was investigated by a more detailed analysis of items that appeared to be good indicators of the AD and PA factors (GLS analysis). This showed that the DSM-III item "lost interest" (Group 4) discriminated poorly between the two factors and that "Crying spells" might be a good replacement in a new set of nine depression indicators. Eight good PA indicators were also identified (the eight listed above).

Normality testing was carried out for the above subset of nine AD and eight PA items (section on testing of normality assumptions). This analysis supported the notion of underlying normal processes for these items and factors. In analyses that allowed non-normal latent response variables (the sections on the analyses of variables) the same factor interpretation resulted as in the regular analysis assuming normality. Thus, there was nothing in these analyses that contradicted the interpretation of rareness of depression and anxiety symptoms as arising from continuous variables that are normally distributed in a normal population with a symptom being reported when such a normal variable exceeds a threshold.

Classification of individuals was carried out in the factor analysis framework by means of estimated factor scores (the classification of individuals). The use of factor scores based on the nine new AD items was contrasted to the DSM-III depression classification. It was found that a significant number of individuals with high factor scores were not classified as depressed.

The Durham data showed a factor structure that was similar to that of Baltimore (comparative analyses section). The same four factors were identified (Table 6). In terms of individual indicators, "lost interest" was again found to discriminate poorly between depression and anxiety. A simultaneous analysis of Baltimore and Durham supported the notion of invariant factor structure. The major difference between the two sites was found to lie in the levels of the factors, with Durham exhibiting lower levels of Anxious Depression and Phobic Anxiety.

NOTE

1. I am indebted to William Eaton for the substantive interpretation of the factors.

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Bengt O. Muthén is Professor at the Graduate School of Education, University of California, Los Angeles. He is currently president of the Psychometric Society. His research interests are in the analysis of categorical data, factor analysis, and structural equation modeling. He is the author of the LISCOMP Statistical Computer Program.