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CHAPTER 1

ANALYSIS OF LONGITUDINAL DATA USING LATENT VARIABLE MODELS WITH VARYING PARAMETERS

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Editors' Introduction

Random effects growth models are currently an exciting topic in quantitative methods (Longford, 1989; Raudenbush & Bryk, 1988; Rogosa & Willett, 1985b). In these models one simultaneously fits individual growth models, estimating parameters for curves of change for individuals, and models for the group(s), estimating parameters that indicate the "typical" change in a classification of people. The idea that researchers should make this kind of distinction among individuals, types of individuals, and groups is one that Ledyard Tucker presented (and excited with) back in the days when the Harris book on methods for studying change aroused the field of developmental methodology. Chapter 1 by Muthén is a good representation of this kind of model. Important improvements and additions to such models are also provided.

This chapter has several particularly noteworthy features. First, Muthén gives an example of the use of one of the programs for random effects

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growth models, VARCL (Longford, 1989), which will be useful to substantive researchers who wish to apply this new technique in their own work. Second, Muthén's incorporation of random effects growth models into the framework of latent variable modeling furthers the understanding and utility of both procedures. Third, Muthén shows how to perform random effects growth analyses using LISCOMP (Muthén, 1987), a widely available latent variable modeling program. (Control language for this is included in an appendix to the chapter.) An advantage of LISCOMP is that one need not assume that the variables of the analysis are multivariate normal. The analyses Muthén proposes thus are directly relevant for research on change; they are likely to be useful in many future studies.

The models Muthén discusses are for interindividual differences in intraindividual growth, which is precisely what many contributors to this book state we must carefully analyze if we are to understand change better. It is interesting to compare and contrast Muthén's models with those of Browne and DuToit (chapter 4) and Meredith (chapter 10). In some ways this chapter is an application of Nesselroade's thinking (chapter 6).

This chapter attempts to bridge two different traditions in the analysis of longitudinal data to describe individual differences in change: random effects modeling and structural equation modeling. Random effects modeling uses both fixed and random parameters and describes the data as T replicated observations on p variables. A primary focus is on differences in parameter values across individuals. Conventional structural equation modeling provides fixed effect techniques and describes a set of p variables observed at T time points by means of a model for pT variables. A primary focus is on differences in parameter values across time. Conventional structural equation modeling has been criticized for being insensitive to individual differences in change (see, e.g., Rogosa, 1987). The possibility of incorporating random effects into the framework of structural equation modeling is considered here. The aim is to provide modeling that combines the special strengths of each tradition.

A REGRESSION MODEL WITH RANDOM EFFECTS

Consider the model for $i = 1, 2, \ldots; I$ individuals observed at $t = 1, 2, \ldots; T$ time points

$$y_{it} = \alpha_i + \beta_i x_{it} + \epsilon_{it}, \tag{1}$$

$$\alpha_i = \alpha + \gamma_\alpha z_i + \delta_{\alpha i}, \tag{2}$$

$$\beta_i = \beta + \gamma_\beta z_i + \delta_{\beta i}, \tag{3}$$

where y, x, and z are observed variables, α, β, and γ are (fixed) parameters, and ϵ and δ are random errors, adding the further parameters $V(\epsilon)$, $V(\delta_\alpha)$, $V(\delta_\beta)$, and $Cov(\delta_\alpha, \delta_\beta)$. Whereas y and x vary over both individuals and time, z is a time-invariant variable describing differences between individuals.

In terms of studies of growth, the x variable is often a time-related variable, such as age. In studies of growth it is natural to assume that the mean of the x variable increases with time, inducing an increase in the mean of the ys. The preceding model covers the special case of straight-line growth with across-individual variation in initial status α_i and growth rate β_i.

Analysis of the type of models outlined in Equations 1–3 can be handled by computer software such as VARCL (Longford, 1989) and HLM (Raudenbush & Bryk, 1988), where the parameters of α, β, and γ; the variances of ϵ and δ; and the covariances among the δs are estimated by maximum likelihood analysis. These programs also provide empirical Bayes estimates of each individual's regression coefficients, that is, the estimates of the α_i and β_i in Equation 1. Such an analysis recognizes that longitudinal data are obtained in a hierarchical fashion, with correlated observations obtained for independently observed individuals. Analogous data structures are found in educational data with students observed within classrooms or schools. The latter application has given rise to the name "multilevel" analysis for such situations (see, e.g., Bock, 1989b).

An Example

Muthén (1983) analyzed the stability of neuroticism measures. These data were obtained as a random sample of Canberra, Australia, electors interviewed 4 times at 4-month intervals in 1977 and 1978 (see Henderson, Byrne, & Duncan-Jones, 1981, for further details). Complete data were available for 231 individuals. The indicators at each time point are four dichotomous items intended to measure "neurotic illness," arising as the yes or no answers to the following questions: "In the last month have you suffered from Anxiety? Depression? Irritability? Nervousness?"

Four variables provide a measure of "life events" (L) that the respondent has experienced in the 4 months prior to each interview. The Neuroticism (N) scale from the Eysenck Personality Inventory is used to measure long-term susceptibility to neurosis. N is here taken as the average score from Occasions 2 and 4. Sex is also recorded. Table 1.1 gives descriptive statistics. For simplicity, a neuroticism score is created as the sum of the four yes or no responses.

In contrast to the aforementioned growth perspective, this example is such that no growth or other mean trend is expected in either y or x. Neuroticism is expected to go both up and down across time. (Note, however, the downward trend in Table 1.1.) The object of the longitudinal analysis in this case is not growth but change or stability. However, the statistical framework of a regression model with random effects is still useful.

Table 1.1
NEUROTICISM (N = 231) MEAN SCORES AND VARIANCES

	Time 1	Time 2	Time 3	Time 4
Neuroticism				
Mean score	1.17	0.81	0.78	0.75
Variance	1.65	1.29	1.24	1.17
Life events				
Mean score	3.86	3.17	2.58	2.42
Variance	6.54	5.89	4.90	5.27
N scale				
Mean score	9.31			
Variance	20.66			

In the neuroticism example the time-varying x variable of Equation 1 corresponds to the life event variable L. The time-invariant z variable of Equations 2 and 3 corresponds to the N scale and sex. Randomly varying intercepts over individuals implies that without stressful life events, different individuals are expected to have different levels of measured neuroticism. The differences in such levels can to some extent be explained by the individual-specific N and sex variables. Variations in the slopes across individuals implies that the L variable has different predictive strength for different individuals. For example, a person with a high N score may react more strongly to stressful life events, corresponding to a positive γ_β.

To illustrate, I shall analyze the neuroticism data by the random coefficient regression program VARCL (Longford, 1989). I will apply a model with random variation in both intercepts and slopes and attempt to explain this parameter variation by N and sex, where sex is a 0/1 variable scored as 1 for female subjects. The results are as follows for a sequence of increasingly complex models. The model number is prefixed by the letter R for random parameters.

Model R1

The first model specifies a random intercept and fixed slopes. No time-invariant, individual-specific variables are used. The VARCL maximum likelihood estimate of the intercept variance obtains a clearly significant value.

Model R2

The second model is the same as Model R1, except that the intercept variation is expressed as a function of N, in line with z of Equation 2. In Equation 2, the regression of the individual intercepts on N (or z) obtains a positive slope estimate of 0.107, with a standard error of 0.011. This regression coefficient is therefore significant. The remaining intercept variance is still significantly different from zero.

Model R3

The third model is the same as Model R2 except that sex is added to explain the intercept variance. The estimated slope in the regression of the intercepts on N is 0.106, with a standard error of 0.011, and the sex slope is 0.099, with a standard error of 0.101. We conclude that N is still a significant predictor, but sex is not. The residual intercept variance is still significant.

Model R4

Model R4 adds a random slope specification to the random intercepts and takes the intercept variation to be predicted by N, while no individual-level variable is used to predict the slope variation. The intercept-related slope for N obtains the estimate 0.104, with a standard error of 0.011, and is therefore still significant. The variance of the slopes is estimated as 0.002 and is not significant. Note, however, that the introduction of random slopes in addition to random intercepts adds not one but two parameters—the variance of the slopes and the covariance between the intercept and the slope residuals. A likelihood ratio test of Model R4 versus Model R2 is the appropriate way to test whether random slopes are warranted (given that the absence of influence from sex is accepted). This test gives a chi-square value of 7.75 with 2 degrees of freedom, which is significant on the 5% level, although not on the 1% level. This indicates a need to include a random slope specification in addition to random intercepts as is done in Model R4, although the nonsignificant variance estimate does not appear to reflect this. If there is a slope variation, it is not very large.

Model R5

The fifth model keeps the specification of random intercepts and random slopes. The intercept variation is still described in terms of N. Compared with Model R4, Model R5 adds N as a predictor of the slope variation. Although N is still a significant predictor of the intercept variation, it is not found to be an important predictor of the slope variation. The coefficient in the regression of the slopes on N is estimated as 0.0060, with a standard error of 0.0033. The ratio of 1.82 does not indicate significance on the 5% level. The estimated mean of the slopes is 0.036 with an estimated variation in the slope of 0.037 (the variance of N is

20.66). According to this model, N explains only 2% of the slope variation. If the nonsignificance is ignored, the positive slope estimate indicates that increasing N increases the strength with which L predicts the neuroticism score.

Model R6

The final model maintains Model R5 features except that sex replaces N as predictor of the slope variation. Sex is not found to have a significant influence, however.

Given this sequence of model estimation and testing, Model R4 is deemed to be the most appropriate for the data. Note, however, that Model R2 gives very similar results. Model R4 specifies that the intercepts vary across individuals as a function of N. The slopes also vary, but they are not explained by individual-level variables such as N and sex. Recall that the amount of slope variation appears ignorable. The estimated R4 model can be summarized as follows. The variance in the intercepts explained by N is estimated as 38%. The estimated intercept variance is 0.59, and it is of interest to compare this variance to that of the dependent variable of the neuroticism score. The estimated variance in the dependent variable at each of the four time points is 1.37, 1.35, 1.34, and 1.35, so that the variation in intercepts across individuals corresponds to about 44% of the total variation.

STRUCTURAL EQUATION MODELING OF REGRESSIONS WITH RANDOM INTERCEPTS

Structural equation modeling of longitudinal data such as the data analyzed above considers a "strung out" data vector for pT variables. In terms of the neuroticism data, the one y and one L observed at four time points then gives eight variables, to which the N and sex variables are added. Interestingly enough, it turns out that such an approach can in fact be used to analyze a model such as that of Equations 1 and 2 (a random intercept model).

The key to such an analysis is to specify a time-invariant factor

$$\eta_{0i} = \alpha + \gamma z_i + \delta_i \tag{4}$$

corresponding to the random intercept of Equation 2. The random intercept notion may then be reconceptualized. In the neuroticism example, we may instead view η_0 as a person-specific latent predisposition toward neuroticism, which can to some extent be predicted by N.

To capture the model of Equations 1 and 2, the structural equation analysis uses a mean and covariance structure model with equality restrictions on the slopes and residual variances over time, while allowing the means, variances, and covariances of N and the Ls to be unrestricted parameters. The structural modeling path diagram corresponding to such an analysis is given in Figure 1.1. This analysis also provides a chi-square test of fit to the restrictions imposed by the model used in sections 2 and 3 and enables straightforward relaxations of such restrictions.

This technique was applied to the neuroticism data using the LISCOMP structural equation modeling program of Muthén (1987). In line with the earlier results we use the four y variables, the four L variables, and the N variable (nine variables in all). A chi-square value of 59.9 with 29 degrees of freedom was obtained for the random intercepts model, Model R2. Hence, this model does not fit well ($p = 0.0006$) when tested against the model of no structure imposed on the means and covariances of the nine variables. Inspection of the distributions of the variables shows strong skewness as might be expected. The normality assumption, imposed for the residuals in Equations 1 and 2, is therefore not tenable and may explain part of the large chi-square. Browne's "asymptotically distribution free" (ADF) estimator (Browne, 1984; Muthén, 1987) was also applied, but it reduced the chi-square to only 50.0. The sample size of 231 may be too

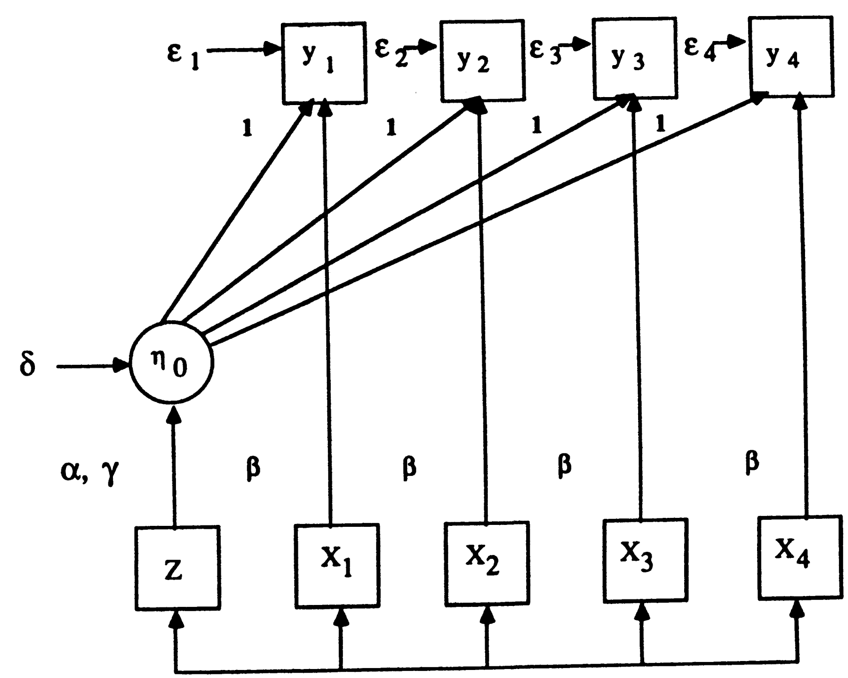

Figure 1.1 Random intercept as a person component in structural modeling.

small to rely on this ADF chi-square value. Below we will work with chi-square differences in models in which it is assumed that non-normality plays a lesser role.

Certain model assumptions can be relaxed in a straightforward fashion in the structural modeling framework. For example, the slopes for L and the residual variances in y need not be equal over time. Table 1.2 describes chi-square statistics for a series of increasingly relaxed models estimated by maximum likelihood. The prefix S refers to structural models. We note that Model S1 is the same as Model R2 (the S counterpart of R1 could be estimated but is not, given the significant influence of N).

The sequence of models in Table 1.2 relaxes the different types of restrictions in the basic model of S1 (R2) in the following order: Equal L slopes over time, equal residual variances in y over time, and uncorrelated residuals over time. For each such model, an alternative specification with cross-lagged influence of L on y is also tested. This means that L at Time 1 is allowed to influence y at Time 2, L at Time 2 is allowed to influence y at Time 3, and L at Time 3 is allowed to influence y at Time 4. R_0^2 refers to the explained portion of variation in the latent predisposition variable η_0, R_1^2–R_4^2 refer to explained variation in the ys, and P_1–P_4 refer to the ratio of estimated variance in the predisposition variable relative to the ys.

Model S1

The basic model of S1 (R2) does not fit well even with lagged L effects. The R^2 and P statistics will, however, be of interest for comparisons with subsequent models.

Model S2

Allowing the slopes for L to be different at different time points gives a significant reduction in chi-square compared with Model S1 (59.95–40.82 with 29–26 degrees of freedom). The drop is also significant if one adopts the lagged model.

Model S3

Model S3 allows both slopes and residual variances to differ across time. This also gives a significant drop in the chi-square level compared with any of the previous models.

Table 1.2
STRUCTURAL EQUATION MODELING

	Model S1		Model S2		Model S3		Model S4		Model S5	
Parameter	Basic	+ lagged L[a]	Unequal slopes	+ lagged L	Unequal slopes and residual variances	+ lagged L	As S3, correlated residuals	+ lagged L	As S4, unequal intercepts	+ lagged L
X^2	59.95	54.32	40.82	38.95	31.75	29.84	28.20	26.73	23.92	21.80
df	29	26	26	23	23	20	20	17	17	14
Probability	.0006	.0009	.0324	.0201	.1055	.0725	.1046	.0621	.1216	.0828
R_0^2	.403	.413	.406	.401	.392	.387	.394	.390	.394	.389
R_1^2	.519	.525	.550	.553	.495	.498	.518	.519	.501	.500
R_2^2	.511	.509	.507	.509	.500	.504	.530	.531	.534	.539
R_3^2	.509	.509	.509	.512	.570	.573	.575	.576	.586	.585
R_4^2	.512	.509	.505	.507	.526	.525	.522	.521	.526	.525
P_1	.443	.452	.434	.426	.388	.381	.408	.401	.434	.427
P_2	.450	.467	.475	.470	.446	.462	.494	.488	.496	.484
P_3	.452	.467	.472	.466	.527	.520	.532	.525	.530	.519
P_4	.449	.466	.476	.471	.494	.486	.490	.484	.492	.484

[a]Life events variables.

Model S4

Longitudinal data would appear to result frequently in residuals that are correlated over time as a result of the effects of omitted predictors. Model S4 tests this, but when the results are compared with those of Model S3, such correlated residuals are found insignificant in this application.

Model S5

So far, the models have implied that the means of the *y*s may differ over time only as a function of mean changes in *L*. In technical terms, the specification of a nonzero intercept in the regression of η_0 on *N* implies equal intercepts in the (reduced-form) regressions of the *y*s on the *L*s and *N*. As a final test, Model S5 relaxes this restriction. Technically, this is done by allowing the intercepts in the equations for *y* at Time points 2–4 regressed on *L* and η_0 to be different from a zero intercept for *y* at Time 1. A significant improvement in model fit is not achieved.

In conclusion, it appears that Model S3 without lagged *L* effects is the most parsimonious model that fits well ($p = .11$). Allowing for differences in *L* slopes and residual variances across time appears warranted. The estimates (and standard errors) of the slopes at the four time points are .125 (.017), .061 (.018), .070 (.020), and .054 (.021), whereas the results for the residual variances are .802 (.087), .660 (.074), .502 (.060), and .592 (.068). Time point 1 appears different from the other time points (this is also seen in Table 1.1). In this model the amount of variation in the intercepts explained by *N* is 39%. The estimated variation in η_0 is 0.62, whereas the estimated variances of *y* at each of the four time points are 1.59, 1.32, 1.17, and 1.25. It is interesting to compare Models R4 and S3 with respect to the estimated ratio of η_0 variance to *y* variance. The η_0 variance is interpreted as variance in predisposition in the structural modeling framework of Model S3 and as intercept variance in the random coefficient framework of Model R4. Whereas Model R4 (which is close to R2 or S1) obtained a ratio of about 44%, Table 1.2 shows that the ratio for Model S3 varies across time and ranges from 39% at Time 1 to 53% at Time 3. The LISCOMP setup for Model S3 is given in the appendix (a LISREL setup would be similar).

STRUCTURAL EQUATION MODELING WITH LATENT VARIABLES

Figure 1.2 shows a path diagram for the structural model for neuroticism of Muthén (1983). Here, multiple indicators are used to capture latent variable constructs in order to avoid distortions of measurement errors. The original four dichotomous

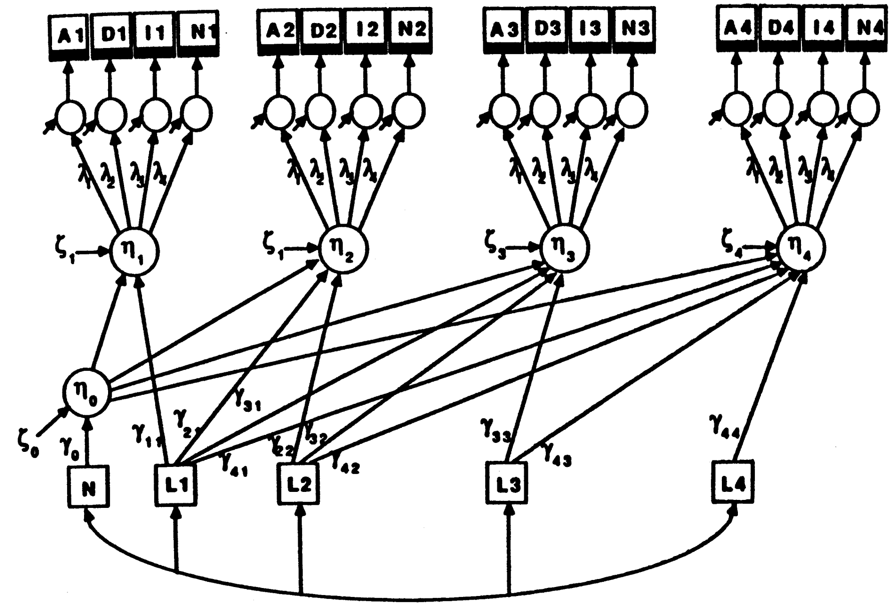

Figure 1.2 Neuroticism model of Muthén. From "Latent Variable Structural Equation Modeling With Categorical Data" by B. Muthén, 1983, *Journal of Econometrics*, *22*, p. 57. Copyright 1983 by Elsevier Sequoia. Reprinted by permission.

variables are used as indicators. Certain features are typical of many structural equation models for longitudinal data:

1. Measurement errors in the outcome variables are of primary concern and multiple indicators are used to define a factor. This gives rise to multivariate response at each time point.
2. The issue of time-invariance of the factor-analytic measurement model used is of primary concern. This assumption, although natural since the same measurement instrument is use, is however not necessary and can be tested.
3. The issue of variations over time in factor means and variances is of interest. The hypothesis of invariance of these parameters can be tested.
4. The specification of how variables correlate across time is of primary concern but little agreement has been reached in the field on how to specify such correlations.

5. Differences in parameters across groups of individuals can be studied by simultaneous multiple-group analysis.

It is interesting to relate these conventional structural equation models to the random effects models. Points 1 and 2 are not of concern in the random effects model, in which a single, error-free outcome measure y is studied. The dependent variable factor in Figure 1.2 takes the role of this y variable.

Point 4 is of particular interest in this chapter. The partial regression coefficient for the path between the factors at two time points was taken to describe the "stability" of the factor by Wheaton, Muthén, Alwin, and Summers (1977), but this idea was criticized by Rogosa (1987) from a growth-modeling perspective. In Figure 1.2, the factors have no such direct effects between them. Instead, the path diagram in Figure 1.2 specifies a latent random variable η_0, which affects each factor with equal weight 1. Muthén (1983) motivated the role of η_0 by generalizing econometric variance component modeling for the pooling of cross-section and time-series data to the latent variable context. As shown above, this component captures intercept variation. The observed indicators may also be correlated as a result of measurement errors' being correlated over time; this could also be added to Figure 1.2.

Random Parameters in Structural Equation Models With Multiple Indicators

Random parameters in structural equation models have been proposed in recent work by Goldstein and McDonald (1988), Muthén (1989), and Schmidt and Wisenbaker (1986). These authors considered data for students observed within classrooms or schools. To clarify the ideas, I shall use a factor analysis model proposed in Muthén (1989). In line with the discussion of regression models with random effects, the notion of classrooms can be translated to individuals and students can be translated to observations across time. As one example, Muthén (1989) considered a factor analysis model in which factor means and measurement intercepts were allowed to vary randomly across classrooms. Again, let i represent individuals (the classrooms of Muthén, 1989), with $i = 1, 2, \ldots, I$, and let t represent the time points (the students of Muthén, 1989), $t = 1, 2, \ldots, T$. For p observed indicators y and m factors η,

$$y_{it} = \nu_i + \Lambda\eta_{it} + \epsilon_{it}, \tag{5}$$

$$\eta_{it} = \alpha_i + \omega_{it}, \tag{6}$$

$$\alpha_i = \alpha + \delta_{\alpha i}, \tag{7}$$

$$\nu_i = \nu + \delta_{\nu i}, \tag{8}$$

where ϵ, ω, and the δs are random residuals, not correlated with each other. Equations 6 and 7 are written to mimic the random intercept formulation of Equations 1 and 2 with η taking the place of y and with no counterpart to x. Equations 5 and 8 express a random intercept model for the indicators of y, with η taking the role of x in Equation 1. With appropriate assumptions, we obtain

$$V(y) = \Sigma_W + \Sigma_B, \tag{9}$$

where

$$\Sigma_W = \Lambda V(\omega)\,\Lambda' + \Theta, \text{ and} \tag{10}$$

$$\Sigma_B = \Lambda V(\delta_\alpha)\,\Lambda' + V(\delta_\nu), \tag{11}$$

where Θ and $V(\delta_\nu)$ may be taken to be diagonal. Σ_W gives the covariance matrix for the p y variables holding the individual constant. Σ_B gives the covariance matrix resulting from variation across individuals.

In line with conventional structural equation modeling for longitudinal data, the analysis vector is of length pT,

$$\mathbf{d}_i' = (y_{i1}', y_{i2}, \ldots, y_{iT}'). \tag{12}$$

The model assumes time invariance (student invariance in Muthén, 1989) of the mean vector and covariance matrix for the p y variables, so that

$$\mu_d' = [\mathbf{1}_T' \otimes \mu_y'] \text{ and} \tag{13}$$

$$\Sigma_d = \mathbf{I}_T \otimes \Sigma_W + \mathbf{1}_T\mathbf{1}_T' \otimes \Sigma_B, \tag{14}$$

where $\otimes$ denotes the Kronecker product, $\mathbf{I}_T$ denotes an identity matrix of dimension $T \times T$, and $\mathbf{1}_T$ denotes a vector of T unit elements.

Assuming multivariate normality and independent observations on $\mathbf{d}$, Muthén (1989) pointed out that this model can be estimated by existing software for structural equation modeling using covariance matrices that are only of order $p \times p$. A simultaneous analysis of two groups is required, in which the maximum-likelihood fitting function may be written as

$$F = I\{\log|T^{-1}\Sigma_W + \Sigma_B| + \text{tr}\,[(T^{-1}\Sigma_W + \Sigma_B)^{-1}\mathbf{S}_B]\}$$
$$+ (I \cdot T - I)\{\log|\Sigma_W| + \text{tr}\,[\Sigma_W^{-1}\mathbf{S}_{PW}]\}, \tag{15}$$

where

$$\mathbf{S}_B = I^{-1}\sum_{i=1}^{I}(\bar{\mathbf{y}}_i - \bar{\mathbf{y}})(\bar{\mathbf{y}}_i - \bar{\mathbf{y}})' \text{ and} \tag{16}$$

$$S_{PW} = (I \cdot T - I)^{-1} \sum_{i=1}^{I} \sum_{t=1}^{T} (\mathbf{y}_{it} - \bar{\mathbf{y}}_i)(\mathbf{y}_{it} - \bar{\mathbf{y}}_i)', \tag{17}$$

where $\bar{\mathbf{y}}_i$ is the individual-specific mean of the p dimensional $\mathbf{y}$ vector taken across time points and $\bar{\mathbf{y}}$ is the p-dimensional vector of total means. The $\mathbf{S}_B$ matrix is a sample covariance matrix for each individual's mean vector, and the $\mathbf{S}_{PW}$ matrix is a pooled within sample covariance matrix, adjusting for individual differences in means. The first group (Equation 16) is viewed as having I observations and the second group (Equation 17) is viewed as having $(IT - I)$ observations. The two groups have certain parameters in common.

We may also note that the mean and covariance structural model implied by Equations 13 and 14 can be viewed as a model for pT variables, in line with the approach to longitudinal data taken by conventional structural equation modeling exemplified above. For example, for three time points we would have

$$\mu_d' = (\mu_y', \mu_y', \mu_y') \text{ and} \tag{18}$$

$$\sum_d = \begin{bmatrix} \Sigma_W + \Sigma_B & & \text{Symm.} \\ \Sigma_B & \Sigma_W + \Sigma_B & \\ \Sigma_B & \Sigma_B & \Sigma_W + \Sigma_B \end{bmatrix} \tag{19}$$

This model can be analyzed in a single-group structural equation model with both the mean and covariance structure imposed on the pT-dimensional variable vector. The appropriateness of the Kronecker structure for the mean vector and the covariance matrix can be tested, even before the factor analysis structure is imposed on μ_y, Σ_W, and Σ_B. Assuming for simplicity's sake a single-factor model, the corresponding path diagram for the covariance structure part of the model of Equations 5–11 could therefore be drawn as in Figure 1.3. This model is clearly identifiable because it describes a standard one-factor model at both the "between" and "within" levels (see Equations 10 and 11). The formulation of random factor means and random measurement intercepts provides a clear rationale for how the factors and the indicators correlate over time. The random factor means part of the model gives rise to the individual-specific δ_α factor in Figure 1.3.

Breaking out an individual-specific component of η_{it}, δ_i, provides a different way to define "stability" than was done by Wheaton et al. (1977). The stability of the vector may be taken as that part of the variance of the factor which is due to δ_i, giving a notion of how much of the factor variance comes from the variation in the personality trait as opposed to time-specific variation. If the time-specific residual component is allowed to have varying variance across time, the stability value will differ over time.

So far in this section, we have assumed stationarity of the factor distributions across time. In growth applications of mixed effects models such as Equations 1 and 2, this assumption is naturally not made. However, in the neuroticism example,

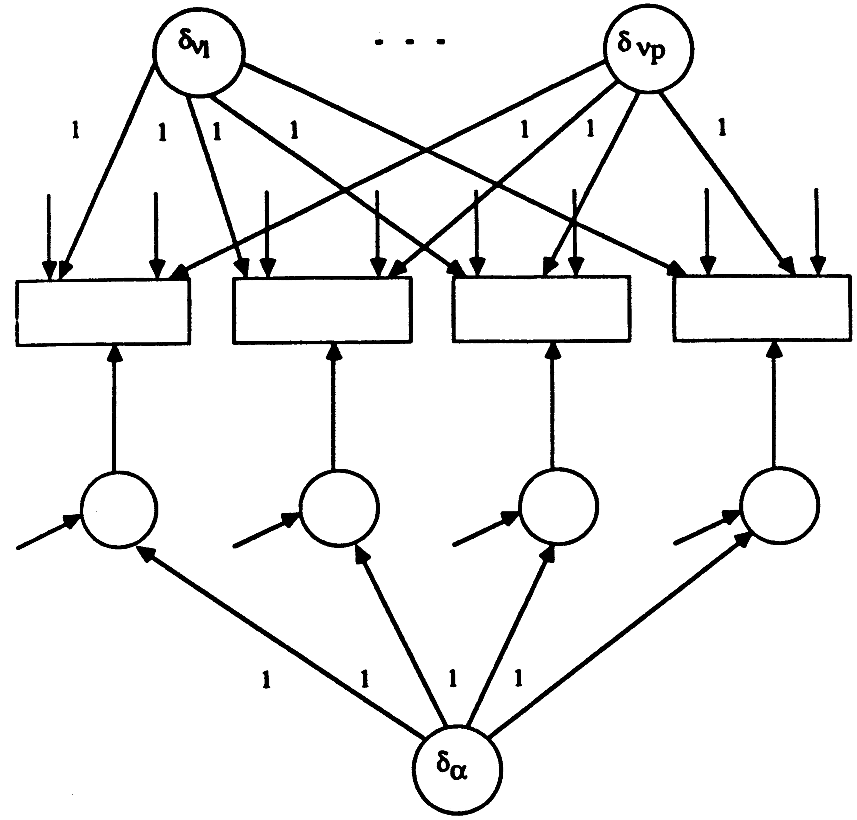

Figure 1.3 Person components of factor and measurement error correlations.

growth over time is generally not expected, but the levels may go up and down in a largely random fashion. Nevertheless, the stationarity assumption may be relaxed and the factors expressed as functions of time-invariant and time-varying observed predictors. This is in line with Figure 1.2 in which the mean of the neuroticism variable is assumed to vary with the life event variable, which in turn is not necessarily stationary in terms of means or variances. Our previous discussion indicates that the path diagram of Figure 1.2 in fact corresponds to a model with random structural regression intercepts for the regressions of η on L at each time point and where this intercept variation is in part predicted by the N variable. Hence, this modeling of longitudinal data does in fact represent a mixed effects model. This particular type of model can be fitted by existing conventional software for structural equation models. Correlated measurement errors could be added to this particular model. As in the study by Muthén (1983), the indicators need not be multivariate normal but can be dichotomous. The LISCOMP program of Muthén (1987) is particularly suited to this task. Simultaneous analysis of several groups of individuals may also be carried out.

It is interesting to compare the results of the neuroticism Model S3 with that of Muthén (1983). The Muthén (1983) model allowed for different effects of *L* across time, as does Model S3, although the residual variances were taken to be equal over time. Cross-lagged effects of *L* appeared small. The percent variation in the predisposition variable relative to the dependent variable factors were estimated at about 68%, a considerably higher value than the 39%–53% range of Model S3. The difference may result from the more proper treatment of the neuroticism measures in Muthén (1983).

This chapter does not address the case of structural models with randomly varying slopes, such as Equation 3. Given this limitation, structural equation software provides more flexibility than is presently given by random coefficients software such as VARCL and HLM. We may easily handle multivariate responses at each time point and formulate factor-analytic measurement models that avoid the influence of measurement errors. Correlated measurement errors over time can also be handled. Multiple-group analysis gives a flexible way to study differences among groups of individuals. Finally, the variables need not be normally distributed.

APPENDIX 1.1

Input to the Computer Program LISCOMP

```
TI  LONGITUDINAL ANALYSIS OF NEUROTICISM
TI  A SINGLE Y REGRESSED ON A SINGLE X AT 4 TIME POINTS
TI  IN A RANDOM INTERCEPT MODEL WITH ONE Z WITHOUT
        STATIONARITY OF X.
TI  NOTE: FIRST 4 ETAS REPRESENT THE YS, NEXT FOUR THE XS,
        THE NINTH ETA REPRESENTS Z (OR N), AND THE TENTH ETA
        REPRESENTS THE LATENT PREDISPOSITION VARIABLE (OR
        RANDOM INTERCEPT) (HENCE AL(5) - AL(8) REPRESENT X
        MEANS)
DA  IY=9 IX=0 NO=231
MO  MO=SE P1 P3 NE=10 NU=FI AL=FI LY=FI PS=FI TE=FI BE=FI
VA  1.0 LY(1,1) LY(2,2) LY(3,3) LY(4,4) LY(5,5)
        LY(6,6) LY(7,7) LY(8,8) LY(9,9)
FR  AL(5) AL(6) AL(7) AL(8) AL(9) AL(10)
FR  PS(1,1) PS(2,2) PS(3,3) PS(4,4) PS(5,5) PS(6,6)
        PS(7,7) PS(8,8) PS(9,9) PS(10,10)
```



```
FR  PS(6,5) PS(7,5) PS(7,6) PS(8,5) PS(8,6) PS(8,7)
        PS(9,5) PS(9,6) PS(9,7) PS(9,8)
VA  0.5 PS(1,1) PS(2,2) PS(3,3) PS(4,4) PS(5,5) PS(6,6) PS(7,7)
        PS(8,8) PS(9,9)
VA  1.0 BE(1,10) BE(2,10) BE(3,10) BE(4,10)
FR  BE(1,5) BE(2,6) BE(3,7) BE(4,8)
FR  BE(10,9)
OU  MN PT ES SE
RA  FO UN=8
(4F2.0,5F3.0)
```