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Analysis of Reading Skills Development From Kindergarten Through First Grade: An Application Of Growth Mixture Modeling To Sequential Processes

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Abstract

Methods for investigating the influence of an early developmental process on a later process are discussed. Conventional growth modeling is found inadequate but a growth mixture model is sufficiently flexible. The growth mixture model allows for prediction of the later process using different trajectory classes for the early process. The growth mixture model is applied to the study of progress in reading skills among first-grade students.

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1 Introduction

This paper outlines how growth mixture modeling can be used to study achievement and learning progress. The work is motivated by a study of reading development among children from Kindergarten to first grade. Section 2 presents the data and the substantive problem, Section 3 discusses random coefficient growth modeling, Section 4 present how random coefficient growth modeling in a latent variable framework can be used to relate the growth factors of two growth processes. Section 5 extends the latent variable framework so that multiple classes of development can be studied.

2 The Substantive Problem

2.1 The Reading Study

The research questions originated from the study Detecting Reading Problems by Modeling Individual Growth (Francis, 1996), also referred to as the EARS study (Early Assessment of Reading Skills). EARS collected data in a modified longitudinal time-sequential design involving about 1000 children. The children were measured four times a year from Kindergarten to grade two. In grade one and two measures included spelling, word recognition, and reading comprehension. In Kindergarten, skills which are considered precursor skills to reading development were measured, such as alphabetic awareness, orthographic and phonemic awareness and visual motor integration. Standardized reading comprehension tests were administered at the end of first and second grade. The background variables gender, SES, and ethnicity were collected.

Francis (1996) focused on the early detection and identification of reading disabled children. In this context, he formulated three research hypotheses: (1) Kindergarten children will differ in their growth and development in precursor skills; (2) the rate of development of the precursor skills will relate to the rate of development and the level of attainment of reading and spelling skills, and individual growth rates in reading and spelling skills will predict performance on standardized tests of reading and spelling; (3) the use of growth rates for skills and precursors will allow for earlier identification of children at risk for poor academic outcomes and lead to more stable predictions regarding future academic performance.

2.2 General Issues

Conventional growth modeling of individual differences in development can in principle use growth trajectory features such as the rate of learning as statistically-based measures of progress. There is a general problem, however, of measuring and modeling student progress over an extended period of time. As the EARS study illustrates, the underlying construct under study in a developmental process is changing and evolving due to maturation of subjects. Reading skills are relevant in first grade but not in Kindergarten. In Kindergarten reading precursor skills are of interest, but lose their relevance in first grade.

This exposes the Achilles heal of growth modeling, namely the assumption that the outcome variable has a constant scale or metric and a stable meaning over time. If it does not, conventional growth modeling is not meaningful. Item Response Theory offers a limited solution to this problem by allowing the formation of scale scores based on different test forms that change over time but have overlapping items. But constructs of interest in a longitudinal study are naturally changing and evolving over time in more fundamental ways and to capture this a more radical solution is necessary.

Changing meaning of the outcome does not make growth modeling impossible. Instead, conventional growth modeling needs to be developed methodologically to suit the research problem. Developmental processes that evolve over time need to be studied in the context of multi-stage growth and multiple processes. There is a need to investigate modeling methodology that can describe how one growth process leads into the next process. It is of interest to see how relationships between trajectories of early growth processes relate to failure/success in later growth processes.

The solution proposed in this paper is essentially to turn the problem into an opportunity. Different developmental phases have different expressions of a construct and should not be forced onto the same scale. Instead, a multi-stage analysis approach should be taken where the different phases are viewed as sequential processes, one leading to another, and are analyzed jointly. This study will focus on how an early process influences a later process as exemplified by how the development of phonemic awareness during kindergarten influences the development of word recognition in first grade. A special focus is on modeling that provides a prediction of a first-grade development by kindergarten development.

3 Growth Modeling

Research hypotheses regarding achievement and learning are often formulated in terms of individual development over time and tested using repeated measurements on groups of individuals. With a developmental perspective, the interest is not so much in the level of a certain outcome at a particular time point as it is in the growth trajectory across multiple time points. Learning outcomes typically show natural systematic growth over time. There may be an initial phase of rapid increase followed by a later phase of leveling out. The starting level, the rate of increase, and the leveling out are of interest in studying learning theories. The focus is on characterizing the individual variation in development and describing it in terms of its antecedents and consequences.

Standard statistical techniques for repeated measures data use random coefficient modeling to describe individual differences in development. This is carried out using software such as BMDP5V, SAS PROC MIXED, and MIXOR using the mixed linear model (see. e.g., Laird & Ware, 1982; Jennrich & Sluchter, 1986; Lindstrom & Bates, 1988), or MLn and HLM drawing on hierarchical linear (multilevel) modeling (see, e.g., Goldstein, 1995; Bryk & Raudenbush, 1992). From a modeling point of view, these approaches are essentially the same. Although it is possible to model multivariate outcomes using these techniques (see, e.g., Thum, 1997; MacCallum, Kim, Malarkey & Kiecolt-Glaser, 1997), applications typically focus on longitudinal development of a univariate outcome variable. Antecedents of individual variation are modeled as time-invariant covariates while time-specific antecedents are modeled as time-varying covariates.

Developmental theories can be better modeled if the analysis methodology can allow trajectory shapes to be of primary focus rather than measurements at specific time points. This means that analysis methodology is needed to describe trajectory shapes not only as outcomes, but also as predictors, as mediators, and, in intervention studies, as the performance of a control group to which the trajectories of the intervention group are compared. Multiple processes, each with its own set of trajectories, for which the interplay and dependencies of the processes are of key interest should also be allowed. The trajectories should be able to have multiple indicators at each time point to reduce measurement error influence and to capture several aspects of the developing construct.

Given this broader research perspective, it is advantageous to perform repeated measures analysis in a more general framework than the mixed linear model or multilevel model. Latent variable structural equation modeling offers such a general framework. While repeated measures analysis of a single outcome variable is obtained as a special case of latent variable structural equation modeling, the generalizations discussed above are possible in the latent variable structural equation modeling framework. This is because the random coefficients are represented as latent variables where the latent variables can have regression relations among themselves and where the latent variables can also represent constructs as outcomes that have multiple indicators. Using psychometric growth modeling introduced by Meredith and Tisak (1990) as a starting point, Muthén and Curran (1997) give an overview of latent variable work related to longitudinal modeling as well as mixed linear modeling and hierarchical linear modeling work and provide an up to date account of the potential of latent variable techniques for longitudinal data suitable for developmental studies. As pointed out in Muthén and Curran (1997), once the mixed linear model is put into the latent variable structural equation modeling framework, many general forms of longitudinal analysis are possible including: mediational variables influencing the developmental process; ultimate (distal) outcome variables influenced by the developmental process; multiple developmental processes for more than one outcome variable; sequential-cohort and treatment-control multiple-population studies; and longitudinal analysis for latent variable constructs in the traditional psychometric sense of factor analytic measurement models for multiple indicators. The latent variable framework also accommodates missing data (see. e.g., Muthen, Kaplan, & Hollis, 1987; Arminger & Sobel, 1989), categorical and other non-normal variable outcomes (see, e.g., Muthen, 1984; Muthen, 1996), and techniques for clustered (multilevel) data (see, e.g., Muthen, 1994, 1997; Muthen & Satorra, 1995).

4 Multi-Stage Growth Modeling of Reading Skills Development using a Conventional Latent Variable Framework

A first attempt at multi-stage modeling of sequential processes uses the conventional latent variable framework for growth modeling. It is suitable for relating multiple outcome variables to each other. The case of a single outcome variable will be discussed first.

4.1 Growth Modeling with a Single Outcome Variable

Consider a certain outcome variable y_j which is measured repeatedly. For individual i at time t, we may formulate the following linear growth model for this outcome variable

$$y_{ijt} = \eta_{ij1} + (a_t - a_0)\,\eta_{ij2} + \epsilon_{ijt};\ t = 1, 2, \ldots, T. \tag{1}$$

Here $\eta_{ijk}(k = 1, 2)$ are latent variables, or growth factors, representing the random coefficients of the growth process, the individually-varying intercepts and slopes, respectively. Furthermore, a_t denotes a time-related variable such as age, a_0 is an anchor point (such as mean age), and ϵ_{ijt} is a residual. The model may be elaborated by adding time-varying covariates to (1) representing educational inputs or other factors influencing the learning at different time points.

The modeling in (1) can be used to address the first research hypothesis of Francis (1996): Kindergarten children will differ in their growth and development in precursor skills. The amount of variation in development is captured by the variance of the growth factors η_{ij1} and η_{ij2}. This variation can be explained by background variables observed for the children, such as gender, SES, and ethnicity. A child's developmental status at a given time is of interest when transitioning to a new phase of learning. Here, developmental status refers to the value predicted by the growth curve, not including the time-specific term ϵ_{ijt} in (1). For instance, if a_0 represents the end of Kindergarten, η_{ij1} represents the developmental status at that time. The child's progress over time adds further useful information. A measure of progress is obtained by η_{ij2}, the linear growth rate for individual i. This describes how the individual reached the Kindergarten end point. A child may have been close to that level throughout the year or may have experienced rapid growth up to that level. Given an estimated growth model for a sample of individuals, a specific individual's status and growth rate may be estimated by Bayesian methods; in psychometrics this is termed factor score estimation. This describes the essence how conventional growth modeling can be used to study progress.

4.2 Growth Modeling with Multiple Processes

The novel growth modeling feature to be considered is relating the random coefficients of the later process to those of the earlier process. This addresses the second research hypothesis of Francis (1996): The rate of development of the precursor skills will relate to the rate of development and the level of attainment of reading and spelling skills and individual growth rates in reading and spelling skills will predict performance on standardized tests of reading and spelling.

Phonemic awareness can be taken as an example of a precursor skill. Consider the influence of phonemic awareness on first-grade word recognition. Using the subscripts p and w to replace the generic j subscript in the growth model of (1), these outcome variables will be denoted y_{ipt} and y_{iwt} with the corresponding subscripts for the η factors. The intercept and slope equations for the growth coefficients of the first-grade process regressed on those of the Kindergarten process may then be written as,

$$\eta_{iw1} = \alpha_1 + \beta_{11}\ \eta_{ip1} + \beta_{12}\ \eta_{ip2} + \zeta_{i1}, \tag{2}$$

$$\eta_{iw2} = \alpha_2 + \beta_{21}\ \eta_{ip1} + \beta_{22}\ \eta_{ip2} + \zeta_{i2}. \tag{3}$$

Here, the β coefficients represent the strength of the dependencies on past performance and acquired skills in transitioning to a new skill. It is assumed that phonemic awareness development predicts word recognition development, emphasizing the importance of the β transition parameters.

As an additional sequential link, the standardized reading and spelling test scores at the end of first grade can be regressed on the growth coefficients of the first-grade process. Letting the reading and spelling scores be denoted y_r and y_s, respectively,

$$y_r = \alpha_r + \beta_{r1}\ \eta_{iw1} + \beta_{r2}\ \eta_{iw2} + \zeta_{ir} \tag{4}$$

$$y_s = \alpha_s + \beta_{s1}\ \eta_{iw1} + \beta_{s2}\ \eta_{iw2} + \zeta_{is}. \tag{5}$$

Products of β coefficients in (2), (3) and in (4), (5) translate progress on precursor skills into predictions of ultimate outcomes on the standardized reading and spelling tests. Background characteristics of the child may have an influence on the dependent variables in all four of these equations.

Assembling the observed variables into the vector $\mathbf{y}_i = (y_{ip1}, \ldots, y_{ipT}, y_{iw1}, \ldots, y_{iwT}, y_{ir}, y_{is})'$ and considering the latent variable vector $\boldsymbol{\eta}_i = (\eta_{ip1}\ \eta_{ip2}\ \eta_{iw1}\ \eta_{iw2}\ y_{ir}, y_{is})'$, (1) may be fitted into the measurement part of a structural equation model,

$$\mathbf{y}_i = \boldsymbol{\nu} + \boldsymbol{\Lambda}\ \boldsymbol{\eta}_i + \mathbf{K}\ \mathbf{x}_i + \boldsymbol{\epsilon}_i. \tag{6}$$

Equations (2) - (5) may be fitted into the structural part of a structural equation model,

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \mathbf{B}\ \boldsymbol{\eta}_i + \boldsymbol{\Gamma}\ \mathbf{x}_i + \boldsymbol{\zeta}_i, \tag{7}$$

where $\mathbf{x}$ represents background variables. The model may be estimated by maximum-likelihood under normality assumptions using standard structural equation modeling software (see, e.g., Muthén & Curran, 1997).

4.3 Results

The growth model in (1), (2) and (3) was applied to the growth processes of Kindergarten phonemic awareness and grade 1 word recognition. Linear growth was found to hold for both processes. A sample of $n = 410$ children had complete data on the four kindergarten measures and the four grade 1 measures and the analyses are based on these children. To capture the phonemic awareness level at exit from Kindergarten, the intercept factor is defined at time point 4. Similarly, the word recognition intercept factor is defined at time point 4 in grade 1.

The maximum-likelihood estimates of the mean of the phonemic awareness slope factor is 0.21. The variance of the intercept and slope factors are 0.64 and 0.02. Both values are significantly different from zero. Their relative size shows the typical feature of much higher level variation than growth rate variation. The correlation between the intercept and slope is high, 0.72. The estimates of the four β coefficients in the growth factor equations (2) and (3) are given in Table 1.

This indicates that for word recognition level at the end of grade 1, i.e. the W

Table 1: Estimates of the Relations between the First-Grade and Kindergarten Growth Factors (standard errors in parenthesis).

Dependent Variable	P intercept	P slope
W intercept		
Unstandardized	0.79 (0.07)	-0.41 (0.40)
Standardized	0.70	-0.07
W slope		
Unstandardized	-0.05 (0.02)	0.32 (0.11)
Standardized	-0.24	0.30

intercept, the phonemic awareness level at the end of Kindergarten (P intercept) is important while the Kindergarten growth rate (P slope) is insignificant. The amount of variation in the W intercept accounted for by the Kindergarten growth factors is 42%. The grade 1 growth rate (W slope) is best predicted by the Kindergarten slope (P slope). In this case, however, only 4% of the variation is accounted for.

5 Modeling with Multiple Trajectory Classes

This section describes shortcomings in the analysis of sequential processes using growth modeling in a conventional latent variable framework. An alternative, extended growth model analyzed in a more general latent variable framework is presented.

5.1 Shortcomings of the Growth Model

The growth model allows for individual differences in development. In this way, the estimated model gives not only an estimated mean curve but also estimates the variation in individual curves as a function of the growth factors. This model allows curves for different individuals to be very different. Nevertheless, the model is restrictive in that it does not recognize that the sample of children may be heterogeneous so that different subgroups may follow different models. This restriction is particularly limiting when attempting to predict a later process from an earlier process.

The use of growth factors as predictors is complicated by the fact that the meaning of a growth factor may be different at different levels of another growth factor. Consider for example the hypothesis that a high Kindergarten phonemic awareness intercept and slope interact to influence good grade 1 word recognition development. The intercept is defined at the Kindergarten exit point so a high positive slope value means that the child has been at considerably lower levels earlier in Kindergarten. This rapid growth can in principle be either good or bad. The rapid growth may be good because the child shows potential for rapid learning that may carry over to grade 1. For example, a low starting point in Kindergarten may be due to detrimental home circumstances but the child grows because its aptitude for reading is good. The rapid growth may be bad because the child has not been at the Kindergarten exit level for long and therefore may have had limited learning opportunities during Kindergarten. It is conceivable that these two alternatives have different plausibility at different Kindergarten exit levels.

If this is the case, the influence of the interaction between Kindergarten intercept and slope is not monotonic and needs a special modeling approach. An approach of this type will now be presented.

5.2 Growth Mixture Modeling

The latent variable model in (6) and (7) will now be modified drawing on the growth mixture model of Muthén, Shedden, and Spisic (1998). This builds on a latent variable structural equation model generalized to K classes of a finite mixture. The heterogeneity of the growth is captured by a categorical latent variable $\mathbf{c}_i = (c_{i1}, \ldots, c_{iK})'$, where $c_{ik} = 1$ if individual i falls in class k and zero otherwise. The modeling and estimation will be presented first, followed by the application to the reading skills development.

5.2.1 Modeling and Estimation

For each class k, continuous outcome variables $\mathbf{y}$ are assumed normally distributed conditional on covariates $\mathbf{x}$, related as follows

$$\mathbf{y}_{ik} = \boldsymbol{\nu}_k + \boldsymbol{\Lambda}_k \, \boldsymbol{\eta}_{ik} + \mathbf{K}_k \, \mathbf{x}_{ik} + \boldsymbol{\epsilon}_{ik}, \tag{8}$$

$$\boldsymbol{\eta}_{ik} = \boldsymbol{\alpha}_k + \mathbf{B}_k \, \boldsymbol{\eta}_{ik} + \boldsymbol{\Gamma}_k \, \mathbf{x}_{ik} + \boldsymbol{\zeta}_{ik}. \tag{9}$$

The covariance matrices $\boldsymbol{\Theta}_k = V(\boldsymbol{\epsilon}_{ik})$ and $\boldsymbol{\Psi}_k = V(\boldsymbol{\zeta}_{ik})$ are also allowed to vary across the K classes. Here, $\boldsymbol{\alpha}_k$ contains the intercepts for $\boldsymbol{\eta}$ for latent class k. The different $\boldsymbol{\alpha}_k$ values are used to represent different trajectory shapes for the different classes.

This is a finite mixture model similar to what has been proposed by Verbeke and Lesaffre (1996). To understand membership composition for the different trajectory classes, it is useful to relate the probability of class membership to background variables. As in Muthen and Shedden (1998), a further component is therefore added to the model, where $\mathbf{c}$ is related to $\mathbf{x}$ through a multinomial logistic regression model for unordered polytomous response. Defining $\pi_{ik} = P(c_{ik} = 1|\mathbf{x}_i)$, the K-dimensional vector $\pi_i = (\pi_{i1}, \pi_{i2}, \ldots, \pi_{iK})'$, and the $K-1$-dimensional vector $\boldsymbol{logit}\ (\boldsymbol{\pi}_i) = (log\ [\pi_{i1}/\pi_{iK}], log\ [\pi_{i2}/\pi_{iK}], \ldots, log\ [\pi_{i,K-1}/\pi_{iK}])'$, this model part is expressed as

$$\boldsymbol{logit}\ (\boldsymbol{\pi}_i) = \boldsymbol{\alpha}_c + \boldsymbol{\Gamma}_c\, \mathbf{x}_i, \tag{10}$$

where $\boldsymbol{\alpha}_c$ is a $K-1$-dimensional parameter vector and $\boldsymbol{\Gamma}_c$ is a $(K-1) \times q$ parameter matrix.

Maximum-likelihood estimation under normality assumptions can be carried out using the EM algorithm. In the EM algorithm, data are considered missing on the latent categorical variable $\mathbf{c}_i$. The complete-data likelihood of the EM algorithm for the model in (8). (9). and (10) considers

$$[\mathbf{c}|\mathbf{x}]\ [\mathbf{y}|\mathbf{c}, \mathbf{x}], \tag{11}$$

where $[\mathbf{z}]$ denotes a density or probability distribution. The first term of (11) corresponds to a multinomial regression with a multinomial latent categorical dependent variable determined by (10), while the second term corresponds to a multivariate normal distribution. $f(\mathbf{y}_{ik}|\mathbf{x}_i) = N(\boldsymbol{\mu}_{ik}, \boldsymbol{\Sigma}_k)$ derived from (8) and (9). The E and M steps of the algorithm are discussed in Muthén, Shedden, and Spisic (1998). A useful side product of the analysis is estimates of posterior probabilities for each individual's class membership,

$$p_{ik} = P(c_{ik} = 1|\mathbf{y}_i, \mathbf{x}_i) \propto P(c_{ik} = 1|\mathbf{x}_i)\, f(\mathbf{y}_{ik}|\mathbf{x}_i). \tag{12}$$

An individual may be classified into the class for which he/she has the highest posterior probability.

In the context of growth modeling the finite mixture model above will be referred to as a growth mixture model. Mixture modeling can be viewed as a form of cluster analysis. Many researchers have attempted to cluster longitudinal measures to capture different classes of trajectories by various ad hoc methods. The present method is a rigorous parametric approach; for related mixture approaches to clustering, see, e.g., McLachlan and Basford (1988). In the present study, a "confirmatory" clustering approach will be used, where parameter restrictions are imposed based on a priori hypotheses about growth. Different prespecified growth shapes can be captured by letting some of the parameters of $\boldsymbol{\alpha}_k$ be fixed. The growth mixture modeling results shown below were obtained using the new latent variable modeling software Mplus (Muthén & Muthén, 1998). Input specifications for the analyses can be obtained from the first author.

The posterior probability computations shown in (12) can be used to derive the most likely class membership for a given individual observation vector $(\mathbf{y}_i, \mathbf{x}_i)$. A typical use is where the estimated model is taken as given and a new individual from the same population is observed. Here, the estimated model is used as a measurement instrument in the sense that an observation vector is translated into a class membership statement.

The Mplus program can be used for such posterior probability calculations holding all model parameters fixed at the estimated values and only doing one E step. Because the estimated model is still valid for a subset of the outcome variables in $\mathbf{y}_i$, posterior probabilities can also be computed using a subset of the repeated measures on $\mathbf{y}_i$ up to a certain time point. This responds to questions of how early a useful classification can be obtained.

The growth mixture modeling approach also provides a way to study early indications of problematic development. As an example, it is of interest to be able to identify students who are likely to belong to Class 1. The estimated posterior probabilities obtained by (12) provides a classification of each individual into the class with the highest probability. This is of interest when using the estimated model to classify a new student as early as possible. In this case, the parameters of the estimated model are taken as given and only the posterior probabilities are estimated. While the model is estimated from all the y and x variable, the estimation of the posterior probabilities can be done using only a subset of early measurements. This is a useful approach to identify children who are at risk for reading failure as early as possible. Muthén, Francis, and Boscardin (1999) provides an analysis of this kind.

5.2.2 Application to Reading Skills Development

Applied to the prediction of first-grade word recognition growth using Kindergarten phonemic awareness growth, $\mathbf{y}_i = (y_{ip1}, \ldots, y_{ip4}, y_{iw1}, \ldots, y_{iw4}, y_r, y_s)'$ and

$\eta_i = (\eta_{ip1}, \eta_{ip2}, \eta_{iw1}, \eta_{iw2}, \eta_r, \eta_s)'$. Here, the modeling includes the standardized reading and spelling test scores y_r and y_s at the end of first grade. These scores are included in the model as two further η variables η_r and η_s that are perfectly measured by corresponding y variables ($\epsilon_r = 0, \epsilon_s = 0$). To illustrate the use of covariates $\mathbf{x}$ in (10), a measure of letters, name, and sounds skills obtained at the beginning of Kindergarten is used. This serves as a proxy for home literacy support and early instruction and is a rudimentary early indicator of both automation of the symbol recognition process needed for deciphering print into language and, in the case of letter sounds, of phonemic awareness/grapho-phonemic awareness.

In (9), the first two elements of $\boldsymbol{\alpha}_k$ contain the means of the phonemic awareness intercept and slope and the next two elements contain the means of the word recognition intercept and slope. The trajectory classes are obtained by fixing the $\boldsymbol{\alpha}_k$ mean of the Kindergarten phonemic awareness intercept and slope to different values. Four classes are chosen to represent variation in both intercept and slope values for phonemic awareness development; they will be described below. The latent class variable is a predictor of first-grade development of word recognition. This is expressed by (9) where the six vectors $\boldsymbol{\alpha}_k$ capture the across-class differences in means. The estimated values of the word recognition intercept and slope means in $\boldsymbol{\alpha}_k$ are of primary interest in the analysis. Given the high number of classes, it is assumed that relatively little within-class variation remains in these growth factors. The variation is instead represented by the latent classes. For simplicity, the latent class variable is therefore taken as the only predictor of first-grade development of word recognition with corresponding zero elements of $\mathbf{B}$

in (9) in the present analysis. The η_r and η_s variables are specified to be predicted by the latent class variable in the sense that their means are allowed to vary across classes, and they will also be predicted by the intercept factors for phonemic awareness and word recognition with corresponding non-zero elements in **B**. The model is shown in path diagram form in the bottom part of Figure 1, where as a comparison the top part represents the conventional growth model estimated in Section 4.

The four prespecified trajectory classes for phonemic awareness are shown in the left-hand panel of Figure 2. Each line is plotted at the mean values of the phonemic awareness intercept and slope for the class. Each class allows variation around this line as a function of variation in the intercept and slope. The classes represent three different mean values at the exit of Kindergarten. These values are determined from the mean and variance of the growth intercept in a single-class analysis of these data, where the intercept is defined at the end of Kindergarten. The values are the mean and plus and minus one standard deviation away from the mean of the intercept growth factor. The slopes for all classes except class 1 are the average values given that intercept value. Classes 1 and 2 differ only in the growth slope, where Class 1 has zero growth. Class 1 is of special interest given that it shows failure in reading precursor development. It is also of interest to contrast Class 1 with Class 2. The choice of four classes is not based on model fit criteria but the degree of separation of classes that is of substantive interest and that can be supported by the analysis. In earlier analyses six classes were used but two classes gave zero class counts when analyzing the full model.

It is of interest to be able to identify students who are likely to belong to the different classes. The estimated posterior probabilities obtained by (12) provides a classification of each individual into the class with the highest probability. In this case, the parameters of the estimated model are taken as given and only the posterior probabilities are estimated.

5.3 Growth Mixture Results

Growth mixture analysis was applied to the same sample as in Section 4.3, except reduced to $n = 409$ due to the inclusion of the covariate. In the analysis, initial specifications of class-invariant parameters are relaxed stepwise to see if a solution could be found with a significantly better fit. Here, fit is evaluated by a log likelihood ratio chi-square statistic obtained for nested models. Growth factor variances for the Kindergarten phonemic awareness intercept and slope was found class-varying with a particularly large intercept variance for Class 4. The residual variances for the standardized reading and spelling test scores were also found class varying.

Table 2 shows the prespecified means for the phonemic awareness intercept and slope for the four classes and also the estimated class probabilities. It is seen that Class 1, showing no Kindergarten growth and a low level at exit from Kindergarten, contains 21% of the children. Class 2, showing rapid Kindergarten growth but the same low level at exit from Kindergarten contains 7% of the children. Class 3 and Class 4 contain children with average level and above average level at exit from Kindergarten, respectively, with 49% and 23%.

Table 2: Fixed Values for the Kindergarten Phonemic Awareness Intercept and Slope Means and the Estimated Class Probabilities.

	Intercept	Slope	Probability
Class 1	-1.40	0.00	.21
Class 2	-1.40	0.20	.07
Class 3	-0.59	0.32	.49
Class 4	0.20	0.43	.23

Table 3 shows the estimated word recognition intercept and slope means for the four classes. The corresponding estimated trajectories are shown in the right-hand part of Figure 2.

Table 3: Estimated Values for the First-Grade Word Recognition Intercept and Slope Means (standard errors in parentheses).

	Intercept	Slope
Class 1	-1.33 (.13)	0.10 (.05)
Class 2	-0.41 (.20)	0.41 (.04)
Class 3	-0.12 (.06)	0.38 (.02)
Class 4	0.95 (.06)	0.26 (.02)

Table 3 and Figure 2 show that children in Class 1 continue to do poorly during first grade in terms of word recognition development. Children in Class 1 do better than children in Class 2. This responds to the earlier discussion about whether rapid growth up to a certain level is better than having been at that level longer. These results indicate that at this Kindergarten exit level rapid growth is preferrable for good first-grade development of word recognition. Children in Class 3 and Class 4 continue to do well in first grade.

The standardized reading test score was found significantly related to the word recognition intercept and slope, but not to the phonemic awareness intercept or slope. The standardized spelling test score was found significantly related to the phonemic awareness intercept and the word recognition intercept. As expected, the estimated means of the two test scores were in increasing order for Class 1, Class 2, Class 3, and Class 4. For both of the two test scores, the Class 1 mean was estimated at approximately one standard deviation below the overall mean.

The estimated multinomial regression of the latent class variable on the letters, sounds, and names covariate is summarized in Figure 3. The mean and variance of this covariate are 0.27 and 0.16, respectively. The figure shows that for students who have covariate values lower than one standard deviation below the mean, Class 1 membership is most likely. For increasing covariate values Class 3 and Class 4, respectively, become more likely.

6 Conclusions

The general growth mixture modeling approach was found to be a useful tool for studying the relationship between two sequential processes. It avoided the complexity of predicting growth in the later process by the growth factors of the earlier process. Instead, a latent class variable with classes corresponding to prespecified growth shapes was used to predict growth in the later process.

Application to predicting first-grade word recognition development by Kindergarten phonemic awareness development resulted in several interesting findings. In particular, it was found that among children with low phonemic awareness scores at the end of Kindergarten, those who had shown little growth during Kindergarten continued to do poorly in terms of word recognition during first grade. An estimated 21% of the children in this sample showed this type of development. The children who had started out lower but had grown rapidly up to this low phonemic awareness level at the end of Kindergarten performed significantly better in terms of word recognition during first grade. An estimated 7% showed this type of development.

The results from the growth mixture analysis may be contrasted with those of the conventional, single-class analysis in Section 4. In the single-class analysis, the slope of the phonemic awareness development was not found to be a significant predictor of the word recognition intercept at exit from grade 1. In contrast, the growth mixture analysis shows that the phonemic awareness slope is an important determinant of word recognition level at exit from grade 1 as illustrated by comparing word recognition development for Class 1 and Class 2.

The growth mixture modeling approach also provides a way to study early indications of problematic development. For example, it is of interest to be able to identify students who are likely to belong to Class 1. The estimated posterior probabilities provide a classification of each individual into the class with the highest probability. This is of interest when using the estimated model to classify a new student as early as possible. In this case, the parameters of the estimated model are taken as given and only the posterior probabilities are estimated. While the model is estimated from all the observed variable, the estimation of the posterior probabilities can be done using only a subset of early measurements. This is a useful approach to identify children who are at risk for reading failure as early as possible. Muthén, Francis, and Boscardin (1999) provides an analysis of this kind.

The line of research described here has important implication for preventive interventions and choice of treatment, i.e. different methods of teaching reading. Children belonging to different trajectory classes may respond differently to a given treatment and the modeling can be used to better assess treatment-aptitude interactions. The modeling can also be used to design different treatments for children belonging to different trajectory classes.

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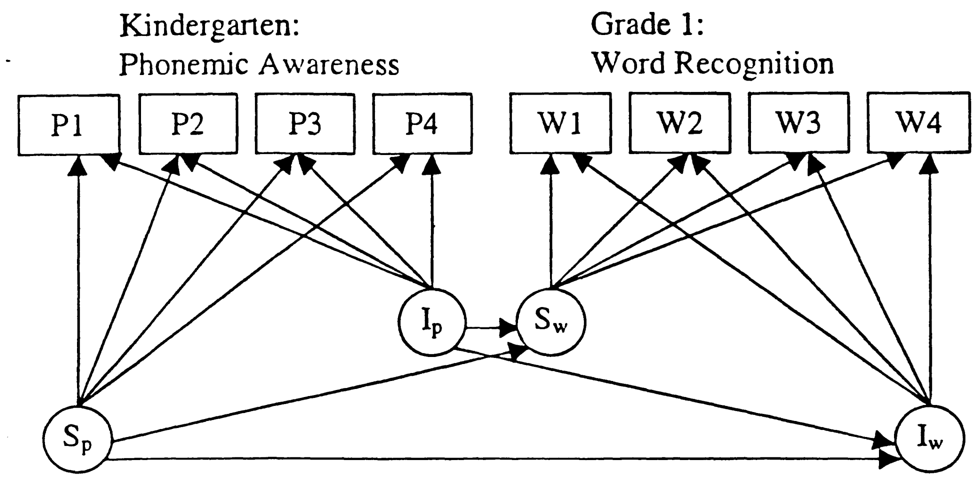
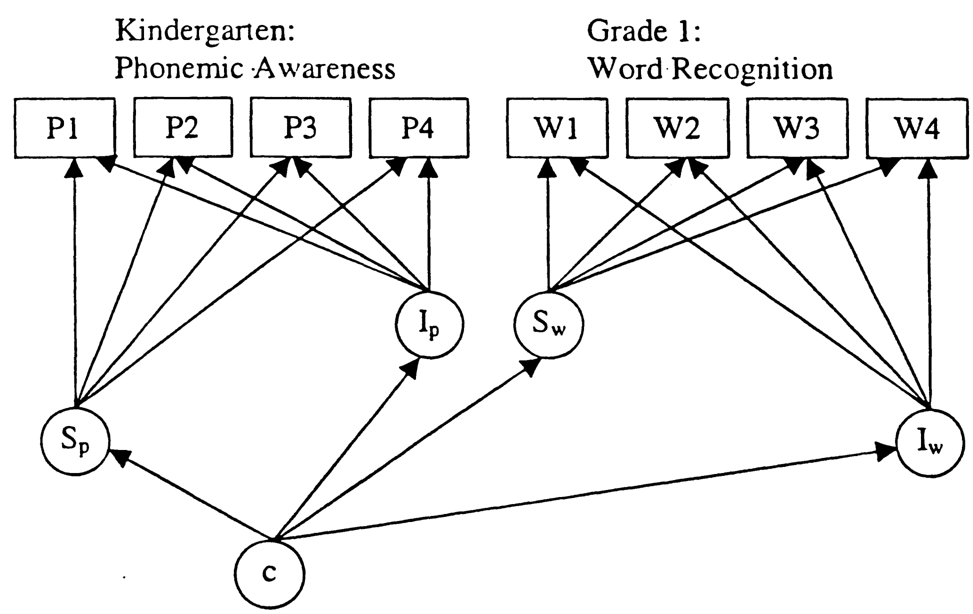

Kindergarten:
Phonemic Awareness
Grade 1:
Word Recognition
P1
P2
P3
P4
W1
W2
W3
W4
Ip
Sw
Sp
Iw


Kindergarten:
Phonemic Awareness
Grade 1:
Word Recognition
P1
P2
P3
P4
W1
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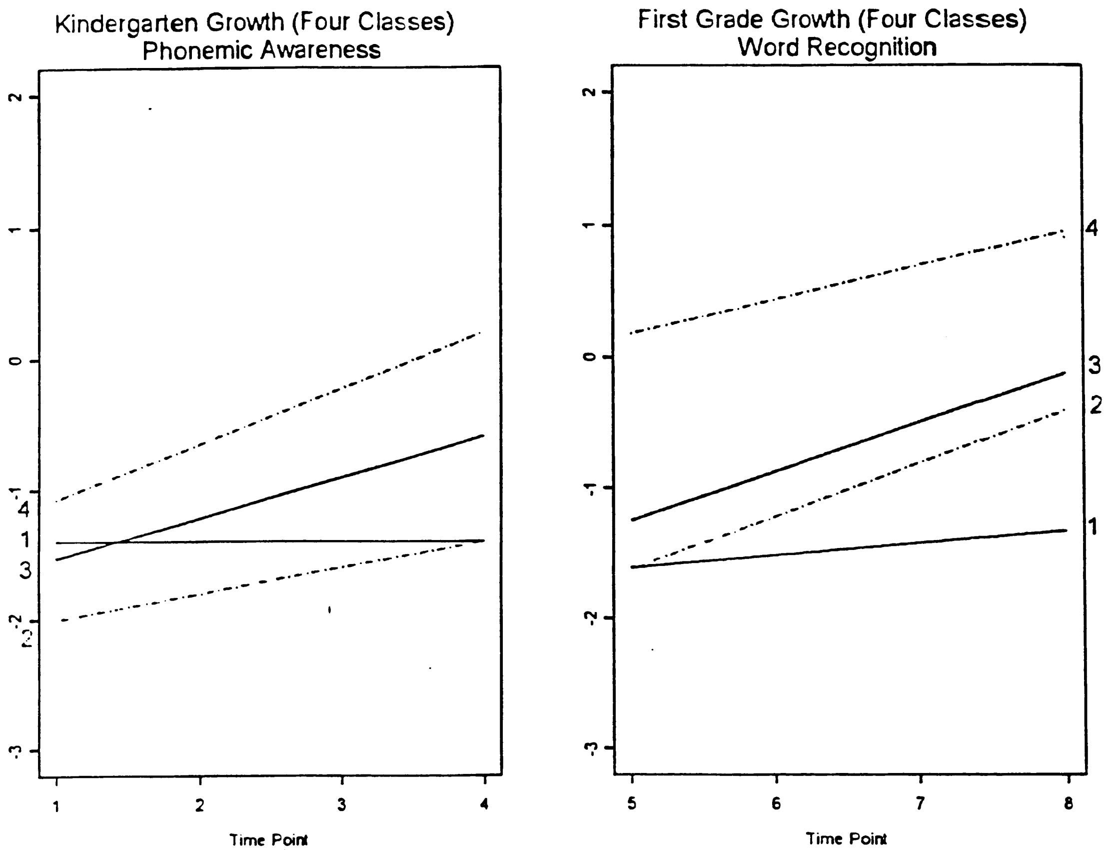

	Class 1	Class 2	Class 3	Class 4
counts	86 (.21)	28 (.07)	200 (.49)	95 (.23)

Letters and Sound as a Predictor of Class Membership