They Don't Show Nothing I Didn't Know: Emergent Tensions Between Culturally Relevant Pedagogy and Mathematics Pedagogy

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This article describes data from the Community Mapping Project, a set of statistical activities and inquiry projects within a summer seminar for high school students. In designing the Community Mapping Project, we attempted to create conditions under which urban students themselves would come to recognize how mathematics is relevant to their lives and their communities. Using mixed methods, we analyzed the preand postassessments and final projects of 25 high school students to investigate what students learned from their experience. We also analyzed the data from video case studies to begin to understand how learning was organized. Our qualitative analysis revealed several tensions that emerged between the goals and norms of our instantiation of a culturally relevant pedagogy and the goals and norms of our mathematics pedagogy. We argue that how these tensions are navigated mediate what opportunity students have for learning statistics. This article provides some considerations and lessons learned that may help inform both teachers who wish to rethink their mathematics pedagogy, and the designers who wish to create culturally relevant curricula.

In light of long standing trends in standardized testing, college enrollment, and current educational policy, the education community has placed an increasing emphasis on ways to improve the achievement of traditionally underserved and marginalized students. Although the field has made major strides toward understanding the complex issues facing schools that serve underserved students, funda-

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mental changes at the classroom level have yet to materialize. It is at this level, the level of the classroom, where the learning sciences community is well situated to contribute to this pressing problem. One proposal has been to teach using the framework of culturally relevant pedagogy (Ladson-Billings, 1995), which recommends three guiding principles for curricular design: (a) the development of a sociopolitical or critical consciousness, (b) a willingness to support cultural competence, and (c) the goal of developing students academically. In this article, we present findings of a DBR project on teaching statistics using culturally relevant pedagogy and discuss how emergent norms and tensions mediated students' mathematics learning.

In designing the Community Mapping Project (CMP), we attempted to create conditions under which urban students themselves would come to recognize how mathematics is relevant to their lives and their communities. CMP provided students with an opportunity to study and produce maps of demographic trends and educational outcomes for their own communities using a computerized mapping tool called a Geographic Information System (GIS). Guided by the three principles of culturally relevant pedagogy outlined previously, we designed a learning environment that would help students develop critical perspectives through openended projects related to their everyday lives and would support the development of students' academic achievement (i.e., mathematics achievement). Although such an environment has the potential to support students' use of mathematics as a tool to document and suggest solutions to social injustices (Gutstein, 2003), the results of our study indicate that specific tensions can emerge, which if not navigated deftly, can have the potential to limit student learning. In this article, we provide some considerations and lessons learned that may help inform both teachers who wish to rethink their mathematics pedagogy and the designers of culturally relevant curricula.

This article presents our findings for two research questions that address learning and the design of learning environments. In our first research question, we investigated if students in this study learned how to use statistics as a tool for social science research and community advocacy. To pursue this question, we focused on two aspects of claims produced by students during a social justice seminar: the extent to which claims about educational inequities were warranted by quantitative information, and the degree to which claims about general trends in the data relied on individual cases or on qualities of the distribution of data. We also examined whether students used statistical measures and tests to back their warrants. In our second question, we investigated how the emergent norms for participation developed, how these norms mediated (i.e., contributed to or inhibited) students' discussions about what counts as adequate evidence, and how these conversations contributed to how well the students learned the core statistical concepts. We believe our second research question is more generative for learning scientists who are interested in designing and studying learning environments for diverse classrooms

in other content domains. Consistent with early iterations of DBR (Brown & Campione, 1996; Cobb, Confrey, diSessa, Lehrer & Schauble, 2003; Design-Based Research Collective, 2003), in this article, we do not intend to settle questions about the complex interactions between subject matter learning, cognition, and culture. Instead, our intention, in this article, is to raise a number of important questions that necessitate debate among the learning sciences community.

In the remainder of this article, we first outline the important considerations for and prominent examples of culturally relevant pedagogy that informed our goals and design. Second, we present the details of our study including who the students were, what the curriculum entailed, and how we collected and analyzed our data. Third, on the basis of pre- and posttests, we present our findings on what mathematics the students learned. In addition, on the basis of students' final presentations, we describe how students came to make arguments using statistics about their community. Finally, in our discussion, we illuminate the tensions that emerged and the lessons learned in how to navigate these tensions. Although we anticipated some of the tensions that emerged, other tensions surfaced only after the seminar ended, and we began to analyze our data.

THEORETICAL FRAMEWORK: CULTURALLY RELEVANT PEDAGOGY

To engage students in meaningful mathematical investigations, we drew on Ladson-Billing's (1995) culturally relevant pedagogy. The project resonated with the three principles laid out in Ladson-Billing's framework—the development of a sociopolitical or critical consciousness, a willingness to nurture and support cultural competence, and the goal of developing students academically.

Critique of Social Inequities

Critiquing social inequities involves developing intellectual habits and perspectives which question and challenge ideologies, policies, and practices by individuals, governments, schools, and other institutions. CMP's aim was to provide a new cultural toolkit for critique and civic discourse that would allow students to find "connected meaning" (Bouillion & Gomez, 2001) and see how mathematics can be used productively to understand and improve their everyday lives. Although there are many routes that can lead students to disengage from school, research shows that one route arises from the lack of connections between academic topics and students' own lives (Atwater, 1998; Barton, 1998; Martin, 2000; Rosser, 1990).

In our planning and conceptualization of this project, we drew on the successful model of Gutstein's (2003) work in Chicago middle school mathematics class-

rooms which demonstrated that social justice issues can foster this type of connected meaning and engagement. Gutstein (2003) showed that minority middle school students who participated in mathematical investigations around issues such as farm workers' wages, the distribution of global wealth, and urban planning, became more engaged. These students developed successful mathematical identities along with a sense of agency and a sociopolitical consciousness. In a similar study, Noguera (2001) organized a learning context in which students discussed inequitable distributions of grades and elective course offerings in their own Northern California community using maps of educational data and regional income. As the project demonstrated, the context generated a high level of engagement and enthusiasm. Although the focus in Noguera's study was not on mathematics, the study reported that students used a substantial amount of mathematics to complete their projects.

Although these studies are impressive because they successfully engaged students in rich academic pursuits, for the learning sciences community, we need evidence that links aspects of social critique to conceptual growth in specific mathematics concepts. Gutstein (2002, 2003) offers evidence of learning by documenting that his students passed their mathematics classes, graduated from the eighth grade, and made significant gains on standardized tests. These are important and significant long-term gains. However, they do not necessarily demonstrate the close connection between learning specific mathematical concepts and engaging in specific culturally relevant activities. In this project, we document the connections between mathematical critiques of education inequity and the development of specific norms of argumentation and statistical concepts.

To make this connection between particular mathematical ideas and particular culturally relevant contexts, we believe we have to look beyond the benefits of engagement and motivation. Following the pattern of Moses and Cobb (2001), we expected that the students' familiarity with the mathematized spaces would serve as a resource for meaning-making by allowing students to draw on their cultural experiences and knowledge to interpret the statistics and displays they produced. For example, in the Algebra Project, Moses and Cobb (2001) used students' familiarity with the transit system and other everyday contexts to help students connect their cultural knowledge to mathematical concepts such as positive and negative integers.

Nurturing and Supporting Cultural Competence

Given the current and historical ways in which the local system of schooling has underserved some African American and Latino youth, we aligned our objective of developing a critical consciousness with the second principle of nurturing cultural competence. We conceptualized nurturing and supporting cultural competence in terms of developing community membership and pride. We wanted students to cri-

tique social injustice, but we also wanted to provide them with a mechanism to contribute to the public discourse and be part of the solution. The hope was that discourse around maps created with census data would help students realize that despite the ways in which their community or cultural group may have been marginalized, as individuals they retained the power to be critical of the system and could author new identities for themselves as agents of change.

Developing Students Academically

Given the opportunities in our present society that are contingent on academic success, one of the most important principles of culturally relevant pedagogy is to provide students with the skills necessary to succeed in schools and other institutional settings as those institutions exist today. Whereas the first principal we discussed—developing a sociopolitical consciousness—aimed at developing students' abilities to critique and change unjust aspects of the system, the principle of academic development focuses on access and achievement. That is, culturally relevant pedagogy simultaneously pursues a pedagogy of access and dissent (Morell, 2004).

For this analysis, we equate academic access and success with learning the key ideas of statistics and developing a robust sense of how to reason and argue with quantitative data. As such, the academic goals of our project were designed to be consistent with the recommendations of the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics (2000) and the California Mathematics Frameworks (1999) which recommend that students learn how to:

- formulate questions that can be addressed with data,
- design experiments, collect data, or use premade data sets to answer questions.
- Organize and display data to answer questions. Students are expected to "select, create, and use appropriate graphical representations of data" (NCTM, 2000, p.176). It is assumed that by engaging in increasingly sophisticated representations of data, students develop increasingly sophisticated ways of reasoning about data (Cobb, McClain, & Gravemeijer, 2003).
- Use appropriate methods to analyze the data. This includes different ways to represent the central tendency of the data (e.g., mean, median, & mode), measures of spread (e.g., standard deviation), the degree to which extreme data points affect different measures (e.g., extreme values have a large impact on the mean but not the mode), and methods to analyze bivariate data and correlations.
 - Generate statistical inferences and predictions based on data.

¹As this article focuses on the mapping and statistics portion of the larger seminar, we do not discuss the instructional objectives for the social science aspect of the seminar.

Conceptual Difficulties With Statistical Reasoning

Despite the importance of statistics, the subject is often difficult to learn because people work with and reason from data in intuitive ways. High school students have at least three relevant difficulties when reasoning with and from data. First, students have difficulty generalizing from data. Second, students' intuitive understanding of central tendency are at odds with some of the procedures used to calculate the mean. Third, when drawing inference about covariation between variables, students often focus exclusively on one aspect of the data.

Generalizing from data. One of the most consistent findings from the literature about statistical reasoning is that students have difficulty reasoning about the aggregate and generalizing from data. Students are inclined to reason from individual, meaningful data points (e.g., their friend's favorite color, or a data point they personally have observed) (Konold & Higgins, 2002). This trend extends to cases in which students compare groups. As late as the eighth grade, students have been found to use individual data points (often opposite, extreme values) to make comparisons between two groups (Bright & Friel, 1998). In this context, this would suggest that our students may well attempt to generalize from the data on their individual school or neighborhood, rather than basing their generalization on the trends in a large number of data points.

Central tendency. Current statistics curricula emphasize computing the mean, and students are often very adept at doing so. However, the literature shows that even students who can compute the mean often do not fully understand central tendency at a conceptual level. Students intuitively describe the central tendency of a group by the "middle clump"—a qualitative version of the mode in which students characterize central tendency based on a cluster of values at the heart of a graphed distribution in which "most" or "the majority" of data points are found in the "hill" of the graph (Cobb et al., 2003). However, for students the mean continues to be a difficult concept to understand throughout high school. Students who can compute the mean do not necessarily use the mean to compare two groups (Pollatsek, Lima, & Well, 1987). Perhaps this is because they recognize that computing the mean removes much of the information they find personally meaningful. Finally, although curricula tend to focus on computing measures of central tendency, at least one program of research has suggested that a better conceptual foundation for understanding how to describe and draw inferences from data is the notion of a distribution (McGatha, Cobb, & McClain, 1999).

On the basis of a series of design experiments that followed the same group of middle school students over 2 years (Cobb, 1999), Cobb and his colleagues recommend that students' learning trajectories should be anchored by the idea of a distribution. From this foundation, they argued that students develop a deeper concep-

tual understanding of the measures of center, spread, relative frequency, and skew (McGatha, et al., 1999). Although GIS maps depict distributions in very different ways than the representational tools used in Cobb's studies, GIS maps do show visually compelling, spatially anchored distributions that are transparent and meaningful to students. The degree to which the two types of visualizations can be bridged in productive ways is an open question.

Covariation. When attempting to reason about the relation between two variables in a data set, students often fail to draw a valid conclusion because they focus on only one aspect of the data. For example, a study by Batanero, Estepa, Godino, and Green (1996) found that when faced with a contingency table about the relation between smoking and Bronchial disease, 70% of a sample of 200 high school seniors were unable to a draw a statistically valid conclusion. The most common error was to only attend to part of the contingency table. The students in this study relied exclusively on the fact that more smokers had the disease than did not, without attending to the number of nonsmokers that had the disease or examining the percentages of the smokers and nonsmokers who had the disease. Other studies have found that students and adults have difficulty in interpreting scatter plots, which is the other main representation for covariation. For example, a study of British nurses found they could not see the relation between blood pressure and age when examining a scatter plot, even though they knew from their experience that a relation did exist (Noss, Pozzi, & Hoyles, 1999). Middle and high school students have similar difficulties (Cobb et al., 2003).

THE SETTING AND PARTICIPANTS

The context for the CMP was a 5-week summer seminar for high school students run by UCLA's Institute for Democracy and Educational Access. The seminar was multifaceted and multidisciplinary. Students carried out social science research around a common theme for which they received Advanced Placement credit in history. In honor of the 50th anniversary of Brown v. The Board of Education, the students studied the changing demographics of Los Angeles schools, the shifting legal policies around integration, and the struggles of communities to realize the promise of education "on equal terms".

Students read classic texts in sociology of education and met with educational researchers and civil rights attorneys. They conducted an oral history project by interviewing community members, activists, and fellow youth. Students also used U.S. Census and California Department of Education data to create GIS maps depicting the transformation of social space in Los Angeles as well as conditions in the schools today. In the process, students moved from being consumers of secondary texts to being producers of critical, public histories that highlighted issues of

ongoing interest to the communities in which they live. At the end of the seminar, the students presented their research at a public forum to UCLA faculty, civil rights attorneys, and members of their own community.

Los Angeles, like many other metropolises, has developed into many relatively racially homogenous neighborhoods. In these neighborhoods, which have specific geographic locations, there is often a correlation between racial boundaries and socioeconomic factors (e.g., income, housing prices, etc.). For example, the central and eastern Los Angeles neighborhoods are predominantly Latino; the neighborhood of South Los Angeles is mainly African American; and the coastal regions are predominantly Caucasian. Income levels are generally greater along the coast. Homogeneous neighborhoods such as these lead to de facto segregation in the schools. Even though there have been voluntary bussing programs since the late 1970's in Los Angeles, it is not uncommon to find neighborhood schools in East Los Angeles that are completely Latino.

Although the Supreme Court decision in Brown v. Board of Education overturned the justification for legal segregation, 50 years after this decision educational inequities persist. In Los Angeles, inequitable distribution of resources within the school system exacerbates the demographic trends that lead to de facto segregation because some schools receive no textbooks or have dilapidated campuses. At the time of this seminar, there was a lawsuit pending in California that argued inequitable conditions and funding existed across the different school districts and subdistricts of the state. However, the inequities alleged in these lawsuits—unequal physical conditions, unequal teacher training, unequal curricular materials—were contested. This controversy set the stage for the students' inquiry projects and analyses.

Statistics and quantitative data were central tools for this larger agenda of making sound social science claims supported by evidence. The CMP was a distinct set of mathematical activities within the larger seminar. In CMP, we integrated mathematics and other disciplinary domains to further foster connections between mathematics and issues of educational inequities. We offered statistics as a tool to help students describe, in quantitative terms, the world they were reading about in their history and sociology texts. The project staff, as well as the design of the activities, focused on apprenticing students into the practices, tools, values, and discourses of statistics and social science research.

To help directly connect quantitative information to social justice issues facing their community, the students learned how to use a GIS called MyWorld (Edelson, Brown, Gordon, & Griffin, 1999). A GIS is a software application which supports visualizations via interactive maps. It uses color, shape, size, and other visual attributes to represent data values that have specific locations on the map. In addition, it is possible to compose GIS maps in layers, which allow them to be superimposed, one on top of the other, like transparency sheets on an overhead projector. For example, one layer might display the population of a region whereas a second

layer might display the schools within the region. Underlying each layer is a fully functional database that allows direct examination and queries. In this case, students were given historical census data (Ethington, Kooistra, & DeYoung, 2000; www.census.gov) and data from California's Department of Education (www.cde. ca.gov). Finally, for any location, the GIS can produce a number of summary statistics and graphs. Figure 1 shows a GIS map produced by a student group during the project. The irregular shapes represent individual census tracts shaded by the percentage of Latinos living in that area. The dots correspond to Los Angeles high schools, with the light colored dots representing schools with less than 80% fully certified teachers. From this map, the students began to explore the quantitative relations between who lives in an area and the qualifications of teachers who teach there.

Opportunities for learning statistical concepts arose during the students' inquiry in small groups. There was no formal, didactic, whole-class statistics instruction. Instead, students learned either by just-in-time instruction prompted by their questions or by engaging in reflective discussions and debates about their claims and

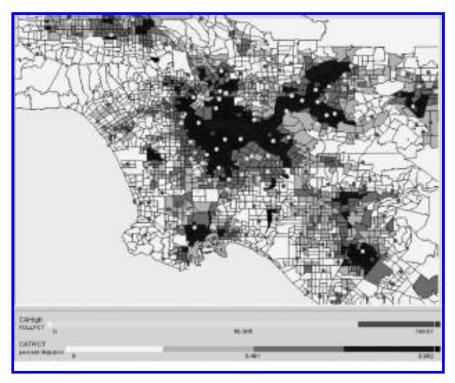


FIGURE 1 GIS map representing percentage of Hispanic population by census tract and the percentage of certified teachers.

evidence. We included several activities that introduced them to the GIS software. In these activities, we asked the students to take on particular roles (e.g., "play a skeptic"). The intention behind the roles was to prompt and encourage productive discussions and debates about the adequacy of evidence. However, some of our planned discussions—involving cross-group debate to facilitate the spread of ideas throughout the whole class—never materialized, and so opportunities to learn varied depending on the particular history of the group.

In addition, the amount of time devoted to the GIS inquiry and quantitative inference is difficult to calculate. All the students spent at least 4 hr using the GIS to complete two mini-inquiry projects designed to familiarize them with the software and the available data. Seven additional days of the seminar included at least 1 hour devoted exclusively to GIS activities. As the projects progressed, the students devoted substantially more time to GIS analysis. However, this more intensive use of MyWorld was often limited to a few students in each group who became the GIS specialists, often taking their work home. We have 46 hr of videotape in which at least one student in a group is working on a statistics related part of the project. Thus, the amount of time any given student spent using the GIS application to create quantitative arguments varied widely but was a minimum of 11 hr of work under the guidance a teacher or researcher and usually substantially more.

Participants

The seminar brought together 25 high school students from across six different high schools in Los Angeles. The schools were selected based on the historical significance of their physical location and the schools' demographics, and they were located in the South, East, West and Central areas of Los Angeles. The schools in South Los Angeles were important because they represented the area that brought the Crawford Case to the California Superior court. This 1968 court case (which the students read) forced Los Angeles to adopt a plan that would counter de facto segregation. We chose the schools in Central, East, and West Los Angeles to illustrate the demographic shifts of an increasing Latino population and the "White flight" from the central areas to the West and North. All the schools had a significant population of African American and Latino youth.

Teachers in each of these six historically low-performing, urban schools recruited the students for this study. The teachers purposely recruited students from a wide range of academic backgrounds. They chose students who would be seniors the following year, who expressed an interest in attending college, who had a chance of qualifying for a 4 year college or university, and who they thought would benefit or be motivated by the experience. Even so, approximately one third of the students had a cumulative GPA of C or lower when they arrived for the seminar. In addition, the students chosen to attend the seminar were all from working class families and primarily self-identified as Latino or African American. There were

five different student research teams comprised of students from different schools. Each research team specialized in a different decade of Los Angeles's history.

Human resources with different levels of assigned and assumed responsibility were present to assist students in conducting their research. Five teachers served as team leaders, one for each of the student research teams. Their responsibilities included helping students formulate their research question and providing students support in executing the series of activities that contributed to their inquiry and formal presentation at the end of the 5-week project.

The teachers were selected based on their demonstrated ability to connect with urban youth, their understanding of social science research, their commitment to integrating social justice into their teaching practice, and the fact that their current teaching position was located in the same areas as the students (but not necessarily the same schools). Four of the teachers taught high school and the fifth teacher, the leader of one of our case study groups, taught elementary school and was a graduate of UCLA's Teacher Education Program. The teachers were all relatively new to the profession, with a range from 1 to 3 years of experience. The teacher who worked in an Elementary school taught all subjects; however, the high school teachers taught different subjects. Among the high school teachers, two taught English, one science and the other social studies.

One week prior to the seminar the teachers and seminar leaders came together in a series of time-intensive meetings to discuss the larger principles of the seminar and what it means to produce a critical public history. In these meetings, the curriculum was open for joint planning and discussion. To help the teachers facilitate inquiry with and around GIS, we planned and practiced two mini-inquiry activities for the 1st week of the seminar and helped the teachers familiarize themselves with the software. During the seminar, the teachers and researchers met on a weekly basis to discuss how the seminar was progressing and to plan the next week's activities.

There were also several other adults present who provided different types of support. There were three researchers, including the first author of this manuscript, who were responsible for organizing different aspects of the seminar. In addition, five graduate students were present twice a week during the GIS activities to videotape the two case study groups and provide general support to students, such as answering technical questions about the GIS software. Toward the end of the seminar, approximately 1 week before the final presentations, eight undergraduate students of color who had previously participated in this seminar began to help students with their presentations and act as successful role models.

METHOD

This article concentrates on two related research questions. First, did the students learn how to use statistics as a tool for social science research and as warrants for their claims about educational inequities? Second, how did the emergent norms for

participation develop and how did they mediate (contribute to or inhibit) students' discussions about what counts as adequate evidence and engagement with core statistical concepts? We approached these questions from the perspective of DBR (Brown & Campione, 1996; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Design-Based Research Collective, 2003). By DBR we mean the project was intended to generate and test theories of learning and instruction by designing a set of activities and tools in which specific elements of the design were anticipated to help students learn domain-specific concepts in specific ways (Cobb et al., 2003). The project reported here was early in the iterative cycle of design, enactment, analysis, and redesign, which is the hallmark of DBR (Design-Based Research Collective, 2003). Therefore, the intent of the project was not to evaluate or perfect the curriculum, but to generate further insight into the complex mechanisms by which culturally relevant pedagogies can and do mediate learning.

Although DBR always involves both prospective and reflective phases in which one is alternately an advocate for one's own design and then one's own critic (Cobb et al., 2003), for this article, we report our reflective analysis of the project. This means our conjectures drove our analysis about how learning is organized and accomplished. Similar to the design of the activities themselves, we developed our conjectures about learning over several iterations during our analysis. At first they were driven by our theoretically based assumptions about how students were "supposed to learn". In later phases of our analyses, our conjectures were refined based on the insights that emerged from preliminary analyses and the contingencies that shaped how the design unfolded.

For these analyses, we employed a mixed set of methods to coordinate three sources of data—written pre- and postassessments, the students' final oral presentations, and video case studies that longitudinally document the history of student activity. The tests and final projects supported assessment of students' statistics learning, and the degree to which they adopted statistical norms for argumentation. Qualitative analyses of the video case studies illuminated tensions that arose between this instantiation of a culturally relevant pedagogy and the project's mathematical objectives. Because of this article's goal of drawing explicit links between culturally relevant pedagogical practices and specific changes in the ways students reasoned with and about statistics, our analyses scored student work higher when it used quantitative evidence as support for claims. This does not imply that we think this style of reasoning is necessarily more advanced than reasoning from qualitative cases—different styles of argumentation have different values and limitations. However, for the purposes of this article, we ranked quantitative evidence higher in our scoring rubric because it was consistent with our instructional goals.

Written Assessments

Due to absences, 21 of the 25 participating students had both pretest and posttests and are included in the pre- and postanalysis. The pretest and posttest consisted of

four questions derived from a clinical interview designed by Konold, Robinson, et al. (2002) that were imported into local social justice contexts. We designed both the pre- and posttests to use the same data set and displayed the data in the same manner (e.g., a table & a distribution plot) for both tests. Although we phrased the questions for both pre-and posttests in the same manner, we couched each test in a different social justice context. For the pretest, the context for the data was the controversy over the eviction of residents of Chavez Ravine to make way for the Los Angeles Dodgers' Stadium. We presented the data set as the income levels of residents in Chavez Ravine and the income of residents in the historical alternative, Pasadena. In the posttest, we provided the students with a Los Angeles Times article which investigated police response times to different neighborhoods. The response times to the two different neighborhoods was the same data set presented to students during the pretest.

We asked the students four questions on both tests.² First, we asked them what type of data (i.e., quantitative or qualitative) they would want to use to evaluate claims about the event. For the second and third questions, we asked them to look at two sets of tabular data, to evaluate a claim, and to evaluate the adequacy of the evidence for that claim. Fourth, we asked them to look at a distribution plot of the same data and again evaluate the claim. To measure the degree to which students would use their new statistical practices to make inferences about novel social justice contexts, none of the questions used GIS displays or a social justice context studied directly by the students. In addition, the time allowed to complete both tests was kept constant. We coded each question of the test using a rubric that targeted two dimensions of reasoning: the extent to which students relied on qualitative or quantitative data, and the extent to which students relied on individual data points or aggregate values. On the basis of our rubric, scores on the written assessments could range from 0 to 16.

Oral Presentations

We used a similar rubric to that of the pre- and posttest to code the final presentations. Although we did not give a score to each presentation, we did analyze each claim that the groups made in terms of the type of claim made and the type of evidence used to support the claim. In coding the type of claim made by the students, we distinguished between describing variables, comparing variables, and making claims about the relation between two variables. The type of evidence used to support each claim involved specific statistics, general references to an area of a GIS map, a particular data point, a narrative story,

²A fifth question was added to the posttest only, but was not included in this analysis to make the tests comparable. The fifth question addressed how students reasoned about correlations between variables based on scatter plots and contingency tables. This was not part of the Konold et al. (2002) interview, but was a conceptual affordability of the GIS software.

etc. Similar to the pre- and posttest, we categorized evidence as either quantitative or qualitative, and based on an individual data point or an aggregate measure. For example, we coded the following statement by a student as a claim about a relation between two variables, using aggregate evidence, but in a qualitative way:

This map is the graduation rate from 1986. The blue dots are [schools in which] more than 70% of the students that graduate. That [refers to the background of the map that shows census data] is around the high income. The red are the less than 30% of graduation rates. As you can see in the low median income—so, you can really tell that they [higher income neighborhoods] have more credentialed teachers.

The example shows the students making a claim about the relation between graduation rates and income levels. It is clear that the students are not generalizing from individual cases but are generalizing about a trend shown over the entire map. Therefore, we coded it as an example in which they were making inferences based on aggregate properties of the distribution. However, although they do quantify one variable (graduation rates), they do not quantify the relation between the two variables (i.e., income & graduation rates). Therefore, we also coded this as an example of the students using quantitative data qualitatively.

Video Analysis

We identified excerpts from the video case studies that illustrated the emergent norms for participation, episodes that illuminated the opportunities for learning, and episodes that revealed tensions between our culturally relevant pedagogy goals and our mathematical goals. Although we expected the navigation of these tensions to mediate the learning outcomes for the students, we also assumed this mediation might develop in unanticipated ways. For pragmatic reasons, we chose the two case study groups because the institutional data collection practices of the California Department of Education and the U.S. Census Bureau had changed over the 5 decades the students were studying, the data available for student projects differed for each group. For example, until the 1970's California Department of Education did not have racial data. The data sets available to the student research teams studying the 1990's and 2000's were the most comparable, so we chose these two groups to be our case studies.

The first step in our qualitative analysis was the creation of content logs of the videotapes during the GIS activities. Content logs provide time indexed descriptions of what happened in the seminar and short analytic notes providing more interpretive descriptions and comparisons based on our theoretical lens (Glaser & Strauss, 1967; Jordan & Henderson, 1995). On the basis of the content logs, we

chose segments of the videotape for closer analysis. The criteria for these segments were: they occurred when the students were preparing their final presentation and they were episodes in which the students produced a claim. To allow us to investigate how students made claims and how the other team members responded to, challenged, or critiqued the evidence for these claims, we produced detailed transcripts for each of these episodes.

For our analysis of these pedagogical tensions, we focused on the norms that were developed, and how they constrained and enabled the students' collective activity. Cobb (1999) differentiated three levels of social organization that pertain to how students grow into their classroom culture or community. At the broadest level are the classroom norms for participation. For example, a typical norm in one of Cobb's teaching experiments is for the students to publicly explain their answers and reasoning. Cobb (1999) goes on to note the connection between understanding and using concepts within a domain, specifically mathematics, and participation in the specialized discourse structures of particular disciplinary content areas. Cobb conceptualizes this level of social organization as sociomathematical norms. Sociomathematical norms address the fact that some answers and justifications are more acceptable for a given community. For example, what counts as an adequate solution for calculating a 15% tip during a dinner conversation may not be an acceptable answer during math class. At the most specific level, Cobb differentiates sociomathematical norms from mathematical practices, which encompass specifics of using tools and procedures to achieve mathematical goals. It is this level that addresses what is most commonly thought of as math, routine ways of interacting with material and conceptual tools to achieve recurring goals. For our analyses, we adopted Cobb's framework and focused on the ways in which language created different normative purposes and sociomathematical norms that served as resources for particular interactions and created opportunities to learn specific mathematical concepts.

FINDINGS

To investigate how well the students learned how to use statistics as a tool for social science research and the degree to which they supported their claims about educational inequities with aggregate, quantitative data, we present data from the written pre- and posttests and our analysis of the claims and evidence used in their final oral presentations. To describe the norms for participation that developed, and how they contributed to or inhibited the type of discussions that research predicts would lead to conceptual growth in this domain, we draw on our video case studies. We argue that several tensions emerged and use our video data to illustrate the tensions and their consequences for learning statistics.

Statistics Learning

Analyses of the pre- and posttest scores show a statistically significant gain of 3.6 points on a scale of 0–16, with the mean score rising from 7 to 10.6 from pretest to posttest (t=5.364, p<. 005). Analyses of the individual items indicate two factors that account for the gains in mean score. First, the most significant factor was the gain on the fourth question, which asked the students to reason from the distribution. On this question, the mean increased from .56 to 2.88 (out of a possible total of 4 points). This was the only question in which the students showed a consistent preference on the posttest for making inferences based on the aggregate data and demonstrated a basic understanding of distributions. There are two possible explanations for this gain. One explanation could be that because the fourth question addressed how students reasoned from the distribution plots, we could attribute the increase to students getting better at reasoning with data presented graphically. Another likely explanation could be that because students had a limited but constant amount of time on both pre- and posttests they simply became quicker at reasoning with data. This second interpretation is consistent with the data in that more of the students answered the fourth question on the posttest compared to the pretest. Still, it seems likely that both of these factors were contributing to the gains in the fourth question, and the overall gain in mean score.

The second factor that could account for the overall gains from pre- to posttest was the trend toward valuing quantitative data over qualitative data. We saw this trend in both Questions 2 and 3. Prior to the unit, the majority of the students talked in qualitative terms about data. After the unit, when answering Questions 2 and 3, the students used quantitative data, but they still often focused on individual data points or the unequal distribution of the samples. The pre- to posttest gains on these two questions were on average .14 and .69 respectively.

However, the modest gains from pre- to posttest do not represent all of the students' mathematical activities during the seminar. The students used a number of mathematical concepts that our assessments did not measure because the assessments focused on statistical instructional objectives. First, they often used algebra to recode existing variables or construct new variables. For example, the census data reports the raw number of each ethnic group living in a census tract. Because census tracts vary in size, however, this did not allow the students to make comparisons. In response, the students constructed new variables that provided the percentage of each group within a census tract and used this new variable for their comparisons. Although the math here appears simple (e.g., creating a new variable based on the count of African Americans in a census tract divided by the total population of the tract), its application in a novel and meaningful context is nonetheless impressive. Second, they did quite a bit of intellectual work to prepare the descriptive statistics they wanted to make inferences from, including choosing the number of breaks for the data display and calculating those breaks. For example, in

Figure 1, notice the display of certified teachers is broken down into two groups—those above and below 80% certified teachers. This was not a random choice but involved a large amount of discussion and manipulations to make a display that was meaningful and showed the trend they wanted. Finally, in their investigations, they were constantly interpreting maps, data tables, graphs, pie charts, bar charts, and histograms to make sense of the data available to them in terms of their research questions. These uses of mathematics, although important, were not the focus of our study and were not measured in any systematic way. This implies that that our written assessment did not capture several additional places in which the students did demonstrate conceptual growth in mathematics.

Use of Quantitative Data as Evidence for Social Justice Claims

One of the first things we noticed in analyzing the students' final presentations was the remarkable consistency with which the students presented their maps. In their presentations, almost all students' claims around the maps followed an informal script, which to our knowledge was not taught or discussed in the seminar, but which the students may have imported from school. The script generally had three parts. First, they explained the legend. Second, students pointed to observations about the map, usually with a phrase such as, "As you can clearly see." This can be seen as taking an initial step toward taking up an academic discourse. Even if it falls short of providing the warrants that would help a person "clearly see," the students are explicitly linking their claims to evidence. Explicitly linking claims to evidence is an important and difficult goal in science and mathematics (Sandoval, 2003). Third, they used a laser pointer or a figural reference (e.g., in the center, on the edge, at the top, etc.) to direct the viewer's attention to the part of the map they were using as evidence.

Recall, claims were coded for whether they used quantitative evidence or whether they used data in nonspecific, qualitative ways. Claims were also coded in terms of how the students made generalizations—based on individual cases or based on aggregate properties of the data set. Finally, claims were coded for their purpose—the description of one variable, the comparison of two variables, or the relation or covariation between two variables.

Following, we provide an example that shows the complexity and level of sophistication of a fairly typical claim. The three distinct subparts of the claim are each underlined and numbered, bracketed text has been added to the transcript to help clarify the student references to the map (which is not included here):

This map shows the total enrollment of the students in the year 1996. Again, the red [census tracts shown on the map] represents the population of African Americans and the blue [census tracts] represents the total population of

Hispanics. All these black dots represent each school, the larger the dots, the larger the concentration of students [total number of students divided by the square footage of school]. See these areas of black dots. These are the areas that the minorities were located [the map shows a cluster of large dots, overcrowded schools in red and blue areas of the map, minority neighborhoods]. And then in the year 2001 [shows a new map], you can see that the population of Hispanics increased¹, and the population of African Americans decreased.² As you can see that the overcrowding did not change, but it just shifted more to the east, where the minorities are still located.³ [The new map shows the large dots in different geographic locations, but they are still clustered in the minority neighborhoods.]

The claim, with its three subparts, was coded as a comparison of two variables, as being based on aggregate properties of the distribution, and as reasoning qualitatively about the data. First, the claim was coded as a comparison because all three subparts compare the value of a variable to its value at another point in time. We coded the claim as being based on aggregate properties of the data because students referred to large areas of the map and did not make generalizations based on individual data points. Finally, we coded the claim as being based on qualitative uses of the data because the students did not explicitly refer to specific quantities or quantify the shifts from one time point to the next. Instead, they used nonspecific, qualitative terms like "increased", "decreased", and "did not change."

In the five final presentations, there were 135 claims. The mean number of claims per group was 20.6 with a range from 18 to 30. Table 1 shows that students cited Qualitative-Aggregate, Qualitative-Individual, and Quantitative-Aggregate evidence at about the same frequency.

It is perhaps surprising that in Table 1 the largest number of claims (58) were made with no evidence at all. However, on closer examination, several factors mitigated the large number of claims made without evidence. First, this level of our

TABLE 1
Types of Evidence While Making Claims and Their Percentage

Type of Evidence	Number (%)
Without evidence	58 (42)
Unsupported narrative claims	21 (15)
Unsupported claim as sub-claim to higher order claim	24 (17)
Other unsupported claims	13 (10)
Qualitative-Individual (e.g., quotations from interviews or personal experience)	27 (20)
Qualitative-Aggregate (e.g., general statements about an area or group)	21 (16)
Quantitative- Individual data points (e.g., specific values for a specific data point)	4 (03)
Quantitative- Aggregate data (e.g., a quantified statement about a group or an area)	25 (18)

coding scheme did not distinguish between types of claims. A common type of claim that students made was a narrative claim. For example, one group claimed, "Resistance in schools is demonstrated through joining gangs, dropouts, refusing to go to class, and talking back to authority. These students feel rejected from the system. Going to schools with unqualified teachers, outdated books, and inadequate facilities." As the bulk of the claim is about student perceptions and behaviors, it is not clear that this type of claim requires support with further evidence in this context. These types of subjective or narrative claims accounted for 36% of all the unsupported claims.

This still leaves 37 unsupported claims (27.4% of the total number of claims) in cases in which evidence seemed appropriate. However, it is reasonable to expect that in any community there will be other types of claims, besides narratives, that do necessitate support with evidence. Even in academic contexts such as this, we do not expect explicit support for every claim with some sort of evidence, particularly in hierarchically related claims. It is common to have claims supported with subclaims, in which the subclaim becomes evidence for the higher lever claim, and in which—for a given audience—it may be unnecessary or cumbersome to provide additional evidence. We coded 24 of the 37 unsupported claims (in which evidence seemed appropriate) as subclaims. In addition, students usually produced subclaims without evidence in a series of 4 to 5 claims which were all topically related and were presented as illustrations of their larger point. For example, in one case, the students made a claim that young people living in low-income communities do not receive an education that prepares them for a 4-year university. They followed this claim by four unsupported claims that illustrated the larger claim—lack of qualified teachers, lack of materials, overcrowding, and issues of safety. It is a separate question whether or not these unsupported subclaims were in fact missed opportunities to delve deeper into the properties of distributions. From a mathematical standpoint, citing evidence would make these claims stronger and may have led to a deeper understanding of the social justice issues at hand. For example, for the case previously discussed, an examination of the data related to college preparedness could have easily led to an examination of the outliers—the students from these schools that did go on to a 4-year university. These discussions of counterevidence may have been used as a springboard for examining the properties of a normal curve, measures of center, and standard deviation.

However, mathematics was not the only consideration in the seminar or in crafting their presentations and claims. For example, one mitigating factor is that students produced these series of unsupported claims more often during the part of the final presentation that was devoted to describing their oral history project. That is, places in which they failed to cite evidence may have been due to the rhetorical genre they were in, rather than a reflection on the value or depth of understanding they had of quantitative evidence. Finally, there was a large amount of variability between groups in the number of unsupported claims, ranging from 3 to 26. There-

fore, although ideally we may wish the number of unsupported claims to be lower than it was here, we would never expect unsupported claims to completely disappear.

To get a clearer picture of how the students used evidence, we eliminated from our analysis the cases in which there were unsupported claims (n=58) and cases in which the students made claims in the context of a narrative (n=16). Table 2 shows the type of evidence the students used (e.g., Quantitative-Aggregate) broken down by type of claim (e.g., descriptive, comparison, & relations). Looking at the column headed "all claims" in Table 2 shows all three types of claims occurred with similar frequency: 37% for descriptive, 35% for comparisons, and 28% for relations. Further, we categorized the context of the claims—with or without GIS maps. We had predicted the GIS context would help the students make more sophisticated claims and support them with valid quantitative evidence. However, comparing the columns of Table 2 (Non-GIS claims vs. GIS claims) does not show support for this hypothesis. Of the three types of claims, only the frequency of " comparing two variables" varied with the context, with the more sophisticated use of statistical evidence—Quantitative-Aggregate—occurring away from the GIS context in 10 claims compared to the 3 occurrences in the context of the maps. "Describing one variable" (such as the income level or ethnicity of an area) was the only context in which students utilized more sophisticated statistical evidence in conjunction with GIS maps. In this case, the ratio was 7 occurrences of Quantitative-Aggregate codes in the context of maps compared to the 1 occurrence in the absence of a map.

TABLE 2
Type of Evidence the Students Used with Each Type of Claim

Claims by Type & Evidence	Non-GIS Claims 36 (100)	GIS Claims 25 (100)	All Claims 61 (100)
Describing one variable	12 (.33)	11 (.44)	23 (.37)
Quantitative -Aggregate	1	7	8 (.13)
Quantitative - Individual	1	0	1 (.02)
Qualitative- Aggregate	2	4	6 (.11)
Qualitative - Individual	8	0	8 (.11)
Comparison of two variables	14 (.42)	6 (.26)	20 (.35)
Quantitative - Aggregate	10	3	13 (.24)
Quantitative - Individual	0	1	1 (.02)
Qualitative - Aggregate	2	2	4 (.07)
Qualitative - Individual	2	0	2 (.02)
Relation between two variables	10 (.28)	8 (.32)	18 (.28)
Quantitative- Aggregate	1	2	3 (.06)
Quantitative - Individual	1	1	2 (.04)
Qualitative - Aggregate	6	4	10 (.15)
Qualitative -Individual	2	1	3 (.04)

Summary of findings on how students use statistics as a tool in their research. Analyses of the pretest and posttest demonstrate modest advances in the ways students were making inferences based on data. Their experiences as they used GIS to investigate social justice issues seems to have increased their perceived value of quantitative data and in certain circumstances led them to shift to using aggregate measures rather than generalizing from individually meaningful data points. In their final presentations, students produced many well-articulated, sophisticated claims about inequitable educational opportunities in Los Angeles. However, in this context they were more likely to use qualitative data than they were to use quantitative data to support their claims.

Of serious concern to the learning sciences should be the curriculum, as implemented, did not get students to interrogate evidence in such a way that more advanced statistical concepts became necessary. The curriculum was designed explicitly to foster these types of discussions by structuring a cycle of local problem solving, followed by seminar-wide discussion and critique that would lead to revisions at the local level. Our qualitative analysis will show that the local activities did spark and could have easily supported these types of rich discussions. However, the enactment of the activities did not support the realization of rich discussions as anticipated. As a result, none of the 135 claims in the students' final presentations was supported by any type of advanced statistical data analysis or significance tests.

How Norms Shaped Student Learning: Illustrative Video Cases

One of the larger emphases of the project was to help students maintain their cultural integrity while succeeding academically. One way this was instantiated and communicated to the students was in the way the teachers explicitly valued the students everyday knowledge of their community circumstances as legitimate and emphasized that students already knew what they wanted to say because they were "living it". Their task was to figure out how to say it. This emphasis on finding evidence to confirm one's preexisting conclusions eventually became a social norm for the community—one that was valuable for motivation and meaning making but problematic from the perspective of both mathematical inquiry and social justice.

The existence of social norms to construct arguments and evidence that are consistent with students' local knowledge of their neighborhoods, however, does not necessarily preclude the development of sociomathematical norms around adequate evidence that would create opportunities to discuss and learn statistics. In the example following (Excerpt 1), during a whole group discussion, we see how one of the program instructors tried to balance the value of students' local knowledge while at the same time pushing for a discussion of what constitutes adequate evidence. Prior to discussing the excerpt, one of the graduate student researchers had

been leading the activity in which all the groups were reporting the change in demographics across the six different Los Angeles high schools and comparing the census data from 1964 and 2002. After the students had reported their findings, the graduate student brought up the issue of "alternative ways to interpret data." She used the GIS software to generate a map based on the 9th grade graduation rates of whites versus African Americans and Latinos in 2002. While generating the map, she continually talked about her process and modeled her reasoning. She provided evaluations such as "once again we don't have very good data," attempted to highlight "the difference between comparing numbers and comparing percentages," and the ways the census data needed to be transformed from "raw numbers" into forms that were relevant to their questions.

Excerpt 1, following, begins when the graduate student researcher stops talking briefly and adjusts the map. During the pause, Patrick, a senior researcher who is one of the three researchers who leads the larger Institute for Democracy and Educational Access seminar, steps in and asks the students if someone wanted to explain the map in "everyday language." The emphasis on the phrase "everyday" was perhaps intentional to counter the mathematical language they had been listening to up to that point. When nobody responded, he rephrased his question, and this time it was taken up by Norma, a Latina student:

Excerpt 1: A seminar leader asks students to interpret a GIS map.

Patrick: What do the colors tell us—the background colors and the dots (..) what do the red and blue dots tell us? Anyone want to try and offer a description in everyday language? (long pause with no response)

Patrick: Let me take half of it. Anyone want to try and describe what is the difference between the darker green and the lighter yellow background colors.

Norma: Oh the—the darker green is white—the rich people

Patrick: Okay [background laughter]

[8 lines omitted]

Norma: And the yellow is Hispanic. I think Hispanic and African American

Patrick: Okay

Norma: Poor [background laughter]

Patrick: So, Norma gave us one set of descriptions. What—what does the actual data say that the dark green is and the lighter colors. Norma was touching on what the data are taken from, but then going off into her own understandings of the neighborhood. You see what I mean. So, remember when Ann started off, the darkest part of the green was the area that had the highest percentage of white people in the census tract, and the lightest yellow, right? Were the areas that had fewest numbers of white people. The fewest percentage in those census tract. So, what Norma was

saying was very similar to that. She was taking some of her own knowledge and putting it into the map as well.

As we see in Excerpt 1, Norma utilized her local knowledge to associate map colors with race even though the map only displayed income levels. Using her local knowledge about where people of different ethnicities live, she began talking about race and income as if they were the same thing. In response to Norma's interpretation, Patrick acknowledges the value of Norma's local knowledge while simultaneously pushing for a more critical look at the data.

In Patrick's response (the last turn of Excerpt 1), we see an example of the tension between the social norm of honoring local knowledge and the sociomathematical norms for critically examining the adequacy of evidence. Patrick is careful to make a distinction between what can be inferred from the data and when Norma is "going off into her own understandings of the neighborhood." However, Patrick does not explicitly mark one type of inference as being the preferred mode for social science research. From our video record of the students' teamwork, it was clear that by the end of the project, this ambiguity contributes to the social norms, such as the value of local knowledge taking priority over the sociomathematical norms that we had intended to foster.

In the example following (Excerpts 2–4), we see a case in which two students begin to raise counterexamples from the data that possibly contradict the group's claim. The raising of counterevidence is one of the types of sociomathematical norms we had designed for and had attempted to develop. This is because one way to convince oneself and others that the counterevidence does not refute the claim one is trying to prove is to go deeper into the statistics of central tendency, correlations, and possibly significance tests. That is, discussing counterevidence was intended to support productive instructional conversations about mathematics.

Although the students raise counterevidence four times in this exchange, the teacher working with the group dismisses their point each time. There may have been many good reasons for the teacher to decide not to pursue counterevidence in this case, not the least of which was that time was running out and the students' final presentation loomed large. However, his choice seems to be consequential for the group. One of his dismissals is an appeal to local knowledge of the area, and at the end of the episode, we see one of the students adopt this line of reasoning and call into question the GIS database using his personal knowledge of Santa Monica.

At the beginning of the example, the teacher is explaining an adjustment he made to the map's display to make the trends that they are examining more visible. He had colored the dots to represent high schools with skewed demographics. Blue dots were high schools with very few African American and Latino students—comprising less than 30% of the total schools' population. Red dots represented high schools in which the Latinos and African Americans represented a large majority of the student population—70% or more. The teacher superimposed the dots

over the average income of each census tract with lighter colors corresponding to lower incomes. Figure 2 presents this map taken from a video still of the shared display (For clarity, Blue dots have been replaced with the white circle symbol, Red dots with the star symbol). This display and discussion is part of the group's overall argument that de facto segregation still exists in Los Angeles.

Excerpt 2: Martha and the teacher work together to relate the map to their thesis.

Teacher: Okay. Go ahead. Go ahead, Martha, you were saying.

Martha: Before blues...like red dots [points to the screen] but now they are all yellow ["red dots" refer to schools with a majority of African American and Latino students; "yellow" refers to low income census tracts].

Teacher: So, a lot of the red dots is more concentrated in the yellow areas

Martha: Yeah.

Teacher: Which kind of goes with our point a little bit better, right? Makes it a little bit more clear, our point?

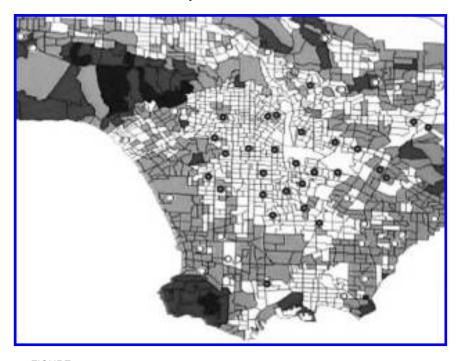


FIGURE 2 Map students were using to display the group's overall argument that de facto segregation still exists (the star symbol represents African American-dominant & Latino-dominant schools, the white circle symbol represents White-dominant schools, background colors represent average income with darker colors showing more wealthy census tracts).

At first, Martha is attempting to understand the changes the teacher had made to the map's color scheme. The colors that had been meaningful to her before are no longer anywhere on the map, and she seems to be struggling to understand exactly what has changed. She also seems to be beginning to see the trend the teacher thought would be more visible with these colors—a relation between income levels and segregated schools. The teacher clarifies, not by explaining the legend, but by highlighting the trend that is now more visible on the map—that "the red dots are more concentrated in the yellow areas." It is interesting to note, their entire exchange only refers to the figural elements of the map and never the referents. That is, they talk of red dots and yellow areas, but not of segregation, ethnicity, and income. Still, the teacher at least seems to think this mapping is transparent to the students despite the changes because he claims this new map helped make their point "a little bit more clear."

Almost immediately after the teacher explained this new display and its purpose, a student again asks what the blue dots represent. This simple request for clarification led the students to examine some of the counterevidence and look more carefully at the high schools with a majority of white students. They began to notice there were a number of predominately white schools in low-income areas.

Excerpt 3: Students raise multiple examples of counterevidence.

Angelica: But what are the blue dots again?

Teacher: So, the blue dots is, there's less than 30 percent Latino and African

Americans in these schools, right.

Angelica: Right.

Teacher: So, for example, if you go to ...

Angelica: But look at the blue spot right there in the low income. [1st mention of

counterevidence

Teacher: This one here?

[omitted]

Teacher: Torrance High.

Angelica: Torrance.

Teacher: Hhh ... Torrance But, but now, but there's just one of those

Liz: No. There's another one. [2nd mention of counterevidence]

[omitted]

Teacher: Santa Monica ... dun dun dun

Liz: There's another one right there.

Teacher: So, we, but see, we click on Santa Monica, right? What do we know about Santa Monica? What do we know about Santa Monica?

Roberto: It's close to the beach

Liz: No, it's not that it's close to the beach its just that, you guys are having problems too, just like the rest of us.

Teacher: No, but what, what do we know about Santa Monica?

Jennifer: That they were the so called diverse

Teacher: (to Roberto) Like do you live in that area?

Roberto: Not anymore.

[omitted]

Angelica: Burbank. [3rd mention of counterevidence]

Teacher: Burbank.

Angelica: So they're like, the blue spot but they are low income too

[omitted]

Angelica: So, there's red ones too, and they're close to browns too ["browns" re-

fers to the higher income census tracts].

Liz: But look right there ... Close to the brown [4th mention of counterevi-

dence]

Teacher: All right. I'm gonna retitle this map. And save it as map. And I'm gonna

show you, and show you the other one.

The shared display allowed students to find counterevidence data points and share them with the group. However, each time the students discovered a "white" high school in a low-income area, the teacher dismissed it. The first time the students provided the counterevidence, the teacher tried to point out there was only one of these outliers. The second time, the teacher did not indulge the use of counterevidence because the specific school—Santa Monica—was in a place where the students did not live. In general, the interpretation for this could be people of color did not live in Santa Monica because none of the students themselves lived there. The third time the teacher acknowledges the evidence, but it is not discussed. Finally, on the fourth time the students used a new tack, pointing out there are red dots (i.e., schools with a larger percentage of African Americans & Latinos) in relatively high-income areas (i.e., darker or "brown" areas of the map). This time the teacher made a bid to close the discussion, moved on to a new map, and ignored the counterevidence.

It is the second move by the teacher in his effort to dismiss the counterevidence that we find the most significant to our analysis. When the teacher asks the students, "What do we know about Santa Monica?" he is appealing to the existing social norm of local knowledge based on the students "lived experience." By giving primacy to local knowledge, the teacher was clearly valuing and nurturing students' sense of community pride in a way consistent with culturally relevant pedagogy (Ladson-Billings, 1995), but at the expense of cutting short a potentially rich mathematical discussion. More important, it comes at the expense of understanding the relation between local knowledge and statistical information. That is, local knowledge can be a very useful resource for generating conjectures, understanding patterns, locating discrepancies, and identifying outliers in the data. It was not our intent that "relevance" should come at the expense of critically examining one's

own experience and understandings. Critical research that investigates social justice requires a critical examination of one's own arguments as well as those of others. Research also suggests that groups who explore each others' ideas are more likely to come to stronger conclusions (Barron, 2003). If the teacher had chosen to pursue the outliers and explore what they knew about those places, it might have complicated the fairly simple social inequity argument that the group seems to have developed up to this point—without disqualifying the argument at all. Further, it would have likely made the mathematics more meaningful in terms of both its relevance and its conceptual depth.

This example of explicitly valuing one norm over another was not common in our data corpus, but did occur several times. However, these events seem significant because over our entire data corpus we see the social norm of producing an argument consistent with their local knowledge being adopted by the students, as is shown in Excerpt 4, following.

In Excerpt 4, one of the senior researchers jumps in before the teacher can move on and attempts to keep the counterevidence thread of the conversation alive and pushes the conversation toward the use of statistics to resolve the issue. However, a student uses his personal experience and knowledge of the local community in question (i.e., he used to live there) to challenge the legitimacy of the data and effectively ends this discussion of counterevidence.

Excerpt 4: Researcher tries to engage the students in a discussion of counterevidence.

Researcher: Before you go to a new map, can I ask you a question about this one [Santa Monica]? See, you guys were talking about whether or not there were blue dots in, in the yellow areas, red dots in the darker areas. But did you look at the, the statistics (for the blue dots)? [makes the computer display tabular data for the school] And so, if you look at the blue dots, you would have.... What do you guys think of this then? What do you—There are exceptions.

Roberto: I should do the Santa Monica (part). It might be (better) because I know that in Santa Monica. The rent is extremely high, and a lot of the—a lot of the traditional families that lived in, lived in Santa Monica for over 51 years had to move away because of the rent and the housing prices.

This series of excerpts from our video case studies demonstrate that some of the productive interactions that we had designed for did emerge, but because the discussions of counterevidence were in tension with the norm for honoring local knowledge, the discussions of adequate evidence were not pursued and did not develop into sociomathematical norms for a critical stance toward evidence in the ways that we had intended. Instead, they blended into an emergent norm which

questioned the validity of evidence when it conflicted with their prior experience. In the journals that students kept, we found many references to "questions you have to ask about data." These references included: "the purpose of the study, who collected the data, what data was collected, when it was collected, how it was collected, and how consistent it was with other sources." This last item on the list resonates with our own qualitative analyses, which showed how students used their local knowledge to trump the data that did not agree with their premise.

Although these questions are legitimate and valuable to ask as a critical researcher, they do not address the inferences drawn from the data, but address inferences about the data set itself. As one quote from a student's notebook says, "The way that data is collected and organized tells us as much information about the people who do the data collection as it does about the phenomenon in question." There is good reason to believe that if a different set of sociomathematical norms had developed, these norms would have created more opportunities to discuss the "phenomenon in question." This in turn may have promoted a deeper conceptual understanding of both the statistics (Cobb, 1999; Cobb, Stephan, McClain, & Gravemeijer, 2001) and the social justice issues (Gutstein, 2003).

This attitude toward claims and evidence, illustrated by the case mentioned earlier and common throughout our data, is best summarized from a quote by one of the students. While the students were preparing their final presentation, a researcher asked the group of students, "What do your maps show?" The girl who was responsible for creating the maps said publicly "inequalities" and then under her breath to her neighbor said, "(They) don't show nothing I didn't know. [laughter] I said I don't know nothing I didn't know." There are two ways to think about this finding. First, one could argue that despite the fact that students had learned to value quantitative data to support their claims, they had not appropriated the type of norms for argumentation we were attempting to establish in the class. That is, students had misunderstood or were unwilling to play the evidence-based language game that we as designers were promoting despite our attempts to introduce and socialize students into it. The second way to think about this finding places the onus on us as designers and teachers to create the activities and participation structures so the students had to continuously engage with issues of what constitutes adequate evidence. From this perspective, despite how complicated it may be to develop and implement a culturally relevant mathematics unit, the tensions we found are not inevitable, and the limitations of our findings were due to the path we as the designers chose to navigate.

The discussions of educational inequalities that occurred within the boundaries of the norms that did develop had both positive and negative aspects. That the students were using their knowledge and experience to make sense of the data has many benefits. It legitmatizes their experience as a resource for academic discourse. It helps them make sense of the world around them, and it shows them how the world can be meaningfully quantified. However, the value placed on what they

already knew often sidetracked what would have been productive, critical discussions about what is adequate and convincing evidence for a claim. Giving primacy to local knowledge provided an easy way out of the complicated discussions that had the most potential to help them learn the statistical concepts they were already using implicitly at a deeper level and to engage in more complex and nuanced critical analyses.

DISCUSSION

This project attempted to provide a rich and engaging way for urban students to learn an important area of mathematics based on the principles of culturally relevant pedagogy. Given the overall findings, the project was reasonably successful at creating an environment for motivating students and engaging them in personally and socially relevant topics. The project helped provide the students with ways of reasoning and engaging in public discourses that may increase their voice and participation in a democratic society, while at the same time giving them access to forms of technical and scientific practice that will be relevant to their further education and eventual employment. It was also effective in fostering some basic conceptual change around statistical inference. The pre-post gains showed significant and targeted improvement in statistical reasoning. Further, our analyses of the students' final presentations showed that even though the students did not show a preference for the GIS maps, when they used the maps and other displays of data, they made many well-articulated claims about inequitable educational opportunities in Los Angeles and used quantitative evidence to support their claims. Qualitative analysis of the video case studies demonstrated that some of the productive interactions that we had designed for did emerge. There was an attempt to honor and use students' local knowledge to make the quantitative evidence more meaningful and engaging. However, a productive balance never stabilized as a sociomathematical norm, and in the local interactions in which the work of the project got done, local knowledge was given primacy.

For our project, a three-way tension emerged. Our first goal was to create a set of activities, norms, and practices that facilitated meaningful exploration and discussion of the core statistical concepts. Our project also had a commitment to having the mathematical activities and discussions be motivated by the students and their perceptions of the need, relevance, and value of statistical reasoning and statistical data in service of their larger social justice advocacy. Finally, we wanted to value and build on the students' local knowledge of their communities, and the problems they faced in their own schools as a resource for making abstract statistics meaningful and as a source of conjectures and questions that students could pursue with data. Through our analysis, we came to recognize the ways in which the norms that would foster mathematical discussions were oftentimes in tension

with the norms that highlighted the social and personal relevancy of the projects. This is not to say that these tensions are inherent to this type of unit, or that we could not have done a better job navigating the tensions. Rather, in any unit of this complexity there will be tensions, and we need ways to conceptualize and anticipate the potential tensions as part of our design process. Anticipating the tensions and setting priorities before the unit starts will help prevent us from merely reacting in the moment and may lead us to navigate a more productive course through this complex terrain.

Integrating GIS into the social justice curriculum did help to make the mathematics relevant because students were familiar with the places being mathematized, and the social inequities were personally tied to their own histories. This familiarity, however, led to the tension between competing norms. Although the visual representation of data on a map that depicted a familiar location allowed the students to link their personal knowledge of their communities to the quantitative data, students often approached their inquiry assuming they knew the truth already. This often led them to focus on finding supporting evidence for their claim. This created tension between the academic goals and norms of statistics, which require skepticism toward evidence, and the project's overarching goal of advocating for a position. We had hoped to navigate this tension by creating a set of sociomathematical norms in the seminar that took a critical stance toward each other's evidence, in which the inferences students made would be open to scrutiny by their classmates. We wanted the students to feel united in identifying and organizing to fight injustice, and we wanted their arguments grounded in empirically strong, statistical evidence. However, trying to develop a sense of skepticism toward the arguments made within the community, while trying to develop a close-knit community of social activists from a diverse group of students within a given timeline, proved to be a delicate balancing act.

Thus, one of the main lessons we learned from this study was the importance of a consistent and coherent set of norms and practices that balance and integrate the mathematical goals and the goals of culturally relevant pedagogy. The findings of our study reinforce the body of research that has argued that the classroom culture—the values, rules, roles, rights and responsibilities of the teachers and students, and the way they shape the instructional conversations—are just as important, if not more important, than any technological tool or curriculum. In this case, we had a powerful tool for visualizing social justice issues in quantitative ways. However, in many ways it was embedded in a classroom culture and rhetorical genre that made some of the target statistical ideas unnecessary. The visually compelling maps, combined with students' local knowledge of the issues and community, was accepted as adequate evidence by the emergent classroom community. This led to a use of evidence to confirm preexisting conclusions. As a result, we saw conceptual growth around the statistical ideas they needed and hence talked about, but that growth had a low ceiling. At times, we saw discussions begin that

addressed counterevidence and the adequacy of evidence, but they were not developed. We believe that had these conversations been systematically pursued—as designed—we would have seen a critical epistemology emerge that would have encouraged a deeper examination of both the mathematics and the social issues.

Our experiences suggest several specific revisions and implications. First and foremost, we argue that navigating a productive path through these competing goals and emergent tensions requires a great deal of pedagogical content knowledge (Shulman, 1986). In our case, the teachers did a great job with many aspects of the project, but the fact that none of them were regular mathematics teachers may have ultimately contributed to the limited findings of conceptual growth in statistics. However, although a solid knowledge of the disciplinary content is important, we do not believe content knowledge alone would have been enough in this case. Teachers need pedagogical content knowledge to chart a course that engages students, leverages their existing knowledge (both physical & cultural), and promotes the right types of mathematical discussions. Pedagogical content knowledge is knowing the typical learning trajectories for specific concepts, knowledge of what sorts of discussions and probes are productive to push student thinking on a particular idea, and how and when these ideas apply to everyday contexts.

In this project, the different amounts of pedagogical content knowledge across the researchers, who lead some activities, and the teachers, who engaged in the daily research with the students, led to unevenness in how opportunities were taken up. In many cases, we did not pursue and build on the students' observations (such as when they raised counterevidence) to their fullest potential. In other cases, instead of challenging students' ideas when the opportunity presented itself (such as when students referred to quantitative data shown in their maps in an nonspecific, qualitative way), their arguments were accepted at face value. To us, this indicates not so much a flaw in the main premise of our design, but as an underestimation of the complexity of teaching this type of unit. It is a reminder to us that in our refinements of this unit we must be more careful to systematically consider the ways different goals, disciplines, learning theories, and social structures may interact with one another during the design process. For those tensions that cannot be predicted before the unit begins, we need to develop priorities and heuristics that will help teachers choose productive courses of action in the moment to achieve their desired ends.

It seems clear that a successful implementation of a culturally relevant mathematics unit will require a great deal of content knowledge, pedagogical content knowledge, planning, and continuous reflection—not to mention a commitment to social justice and the education of underserved students of Color. If implemented in a regular classroom, the need for time and activities that address both the social issues and the mathematical content seem to require the integration or coordination of social studies and mathematics classrooms. This would be difficult in a typical high school because of the scheduling practices and the difficulties in finding

the same group of students in both classes, especially with the current practice of tracking in the mathematics curriculum. Implementing this type of unit in a middle or elementary school might make it easier to integrate social studies with mathematics. However, there is the likelihood of exacerbating the issue of pedagogical content knowledge. This indicates that we may not have addressed additional hurdles in this article for teachers and designers who attempt to implement a culturally relevant mathematics environment. At whatever grade level this type of learning environment is implemented, it would require a set of professional development materials and training that concentrated on a teacher's pedagogical content knowledge, including how to identify and pursue mathematically rich conversations and connect them to the students' own lives, local experiences, and interests. In making these connections, the teachers would need to be particularly attentive to opportunities to discuss how statistical evidence may support, conflict with, or modify one's personal experience and assumptions about the world in which we live.

Related to the practical considerations of implementing this type of unit in a more typical classroom is the division of labor that emerged across teachers, aides, and researchers. In some senses, the seminar was unique and somewhat privileged in the sheer amount of human capital it brought to bear to further students' development. There were 3 educational researchers, 5 teachers, 5 graduate students and 10 undergraduate students regularly involved in the seminar activities for 25 high school students. However, with each group of adults came slightly different agendas and understandings of the objectives of the seminar. The diversity of adult perspectives may have contributed to the tensions that emerged in the classroom culture and the inconsistencies in the norms for what constituted adequate evidence. Given that prior to the seminar, there was no shared history and no existing shared repertoire of practices to guide the group's coordinated activity, it seems likely that this negotiation developed over the course of the project. Because the students engaged with a variety of adults, but different combinations of adults depending on the activity, this negotiation process was complicated and unpredictable. Here is one case in which the lack of human resources in the typical classroom may be an advantage. Although logistical concerns would be more difficult to solve in a classroom with one teacher, it is likely that norms and participation structures that develop are more consistent.

Finally, in retrospect, it is clear that there are at least three ways one can interpret the term "relevant" in culturally relevant pedagogy. The first interpretation focuses on familiarity of the content or context of the lesson and borrows these contexts from students' daily lives (Moses, 2001). The second interpretation focuses on the motivational value of a lesson's perceived value to students' lives outside of school (Gutstein, Lipman, Hernßndez, de los Reyes, 1997). The third interpretation focuses on the familiarity of the process and participation structures by which students engage with the lesson, and the degree to which students' existing repertoires for participation are made legitimate in the academic context (e.g., Heath,

1998; Lee, 1993; Tharp & Gallimore, 1989; Warren & Rosebury, 1995; Wortham & Contreras, 2002). One of the limits of the CMP was that it attended to the first two of these interpretations without anticipating the tensions that would arise between the social and sociomathematical norms as we integrated the third interpretation of relevance into the project.

The third interpretation of cultural relevance is to focus on the process rather than the content or purpose of instruction. The premise here is that as learners move across the settings of everyday life, they participate in many different forms of culturally organized activity. In doing so, students develop repertoires of practice well adapted to their own lives (Lee, Spencer, & Harpalani, 2003). The mode of participation they encounter in school, however, may be more or less familiar to them (i.e., it may conflict, complement, etc.) depending on their histories of participation across the various settings of their home and community. Simply put, competencies, knowledge, norms, and values developed outside of school can be relevant inside of school, regardless of whether or not they are consciously and explicitly connected. Instructional interactions can be more or less effective to the degree students can successfully negotiate, leverage, and navigate the multiple practices they already know with the ones they are required to participate in at school.

There are a number of successful examples of this approach (e.g., Heath, 1998; Lee, 1993; Tharp & Gallimore, 1989; Warren & Rosebury, 1995; Wortham & Contreras, 2002). In the arena of literary reasoning, Lee's Cultural Modeling projects (2001a, 2001b, 2003) have leveraged the discourse practice of signifying, which is identified with (but not limited to) a game of ritual insult in African American English (e.g., "your mama is so skinny that..."), to facilitate high school students' abilities to understand literary tropes encountered in literature. The approach is centered around a metacognitive strategy of mapping existing repertoires of discourse to literary discourse. First, students analyze their own discourse to make explicit the modes of thinking and talking they are already using. Lee then used these cultural discourse practices as a model to create classroom participation structures that honor these discursive norms and create roles in which students were positioned as competent text users—interpreting symbols, satires, and irony in everyday contexts—before they formally know what symbolism, irony, or satire is. Finally, the students examine points of similarity between signifying and the use of tropes in literature, with the teacher helping them to add the academic terms and concepts to their exiting repertoire of practice.

In the CMP, we did not overlook discourse in our design. Prior to the unit, we articulated the types of debates about claims, data, and evidence that would lead students to engage with statistical concepts. What we did not consider in enough depth, however, was the trajectory of participation needed to establish the sociomathematical norms and practices for a group of students with diverse academic and cultural backgrounds. Further, we did not fully appreciate how this trajectory

of participation leading toward social justice advocacy would interact with the norms and informal practices of everyday argumentation.

Given the successes and challenges of the first iteration of this project, we think it is clear there is a need for more research in the learning sciences at the nexus of what we know about how students learn, and what we know about how to successfully engage students from nondominant groups. From our experience, we see the value of collaborative teams of researchers with expertise in the disciplinary content, the cognitive processes of learning and instruction, the dynamics of classroom practice, the home cultures and communities of the students, and the history of social justice. These types of multidisciplinary teams hold a tremendous amount of promise both for improving student learning environments and for helping the learning sciences develop a more mature understanding of how culture is intertwined with learning (Lee, Spencer, & Harpalani, 2003). However, along with these diverse research teams will come a competing set of priorities and approaches that can impact what and how students learn. To achieve the promise of culturally relevant learning environments, research teams will have to explicitly discuss the social norms and structures of the learning environment to the same extent that we have traditionally discussed the technologies and activities of our designs.

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REFERENCES

Atwater, M. M. (1998). Science literacy through the lens of critical feminist interpretive frameworks. *Journal of Research in Science Teaching*, 35(4), 375–377.

Barron, B. (2003). When smart groups fail. Journal of the Learning Sciences, 12(3), 307-359.

Barton, A. C. (1998). Teaching science with homeless children: Pedagogy, representation, and identity. *Journal of Research in Science Teaching*, 35(4), 379–394.

Batanero, C., Estepa, A., Godino, J., & Green, D. (1996). Intuitive strategies and preconceptions about association in contingency tables. *Journal for Research in Mathematics Education*, 27(2), 151–169.

- Bouillion, L., & Gomez, L. (2001). Connecting school and community with science learning: Real world problems and school-community partnerships as contextual scaffolds. *Journal of Research in Science Teaching*, 38(8), 878–898.
- Bright, G. W., & Friel, S. N. (1998). Graphical representations: Helping students interpret data. In S. Lajoie (Ed.), Reflections on Statistics: Learning, teaching, and assessments in grades K-12. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences*, 2(2), 141–178.
- Brown v. Board of Education, 347 U.S. 483 (1954).
- Brown, A. L., & Campione, J. C. (1996). Psychological theory and the design of innovative learning environments: On procedures, principles, and systems. In L. Schauble & R. Glaser (Eds.), *Innovations in learning:*New environments for education (pp. 289–325). Hillsdale, NJ,: Lawrence Erlbaum Associates, Inc.
- Cobb, P. (1999). Individual and collective mathematical development: The case of statistical data analysis. *Mathematical Thinking & Learning*, *I*(1), 5–43.
- Cobb, P., McClain, K., & Gravemeijer, K. (2003). Learning about statistical covariation. Cognition & Instruction, 21(1), 1–78.
- Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, 10(1), 113–163.
- Edelson, D. C., Brown, M., Gordin, D. N., & Griffin, D. A. (1999). Making visualization accessible to students. GSA Today, 9, 8–10.
- Gee, J. (2000–2001). Identity as an analytic lens for research in education. Review of Research in Education, 25, 99–125.
- Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, *34*(1), 37–73.
- Gutstein, E., Fey, J. T., Heid, M. K., DeLoach-Johnson, I., Middleton, J. A., & Larson, M. (2005). Equity in school mathematics education: How can research contribute? *Journal for Research in Mathematics Education*, 36(2), 92–100.
- Gutstein, E., Lipman, P., Hernández, P., & de los Reyes, R. (1997). Culturally Relevant Mathematics Teaching in a Mexican American Context. *Journal for Research in Mathematics Education*, 28, 709–737.
- Heath, S. B. (1998). Working through Language. In S. Hoyle & C. Adger (Eds.), *Kids Talk: Strategic language use in later childhood*. New York: Oxford University Press.
- Holland, D., Lachiotte, W., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.
- Konold, C., & Higgins, T. (2002). Working with data: Highlights of related research. In D. Schifter, V. Bastable & S. Russel, J (Eds.), *Developing mathematical ideas: Collecting, representing, analysing*. Parsipany, NJ: Dale Seymour Publications.
- Ladson-Billings, G. (1995). Toward a theory of culturally relevant pedagogy. American Education Research Journal, 35, 465–491.
- Lee, C. D. (2001a). Is October Brown Chinese? A cultural modeling activity system for underachieving students. American Educational Research Journal, 38(1), 97–141.
- Lee, C. D. (2001b). Signifying in the Zone of Proximal Development. In C. Lee & P. Smagorinsky (Eds.), Vygotskian Perspectives on Literacy Research: Constructing Meaning through Collaborative Inquiry (pp. 191–225). MA: Cambridge University Press.
- Lee, C. D. (2003). Toward a framework for culturally responsive design in multimedia computer environments: Cultural modeling as a case. Mind, Culture, and Activity, 10(1), 42–61.
- Lee, C. D., Spencer, M. B., & Harpalani, V. (2003). "Every Shut Eye Ain't Sleep": Studying how people live culturally. *Educational Researcher* 32(5), 6–13.
- Markus, H., & Wurf, E. (1987). The dynamic self concept: A social psychologist perspective. *Annual Review of Psychology*, *38*, 299–337.

- Martin, D. (2000). *Mathematics success and failure among African-American Youth*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc.
- McGatha, M., Cobb, P., & McClain, K. (1999). An analysis of students' initial statistical understandings. Paper presented at the American Educational Research Association, Montreal, Canada.
- Mehan, H., Villanueva, I., Hubbard, L., & Lintz, A. (1996). Constructing school success: The consequences of untracking low-achieving students.
- Moll, L. C., & González, N. (1994). Lessons from research with language-minority children. *Journal of Reading Behavior*, 26(4), 439–456.
- Morrell, E. (2004). *Becoming critical researchers: Literacy and empowerment for urban youth.* New York: Peter Lang.
- Moses, R. P., & Cobb, C. E. (2001). Radical equations: Civil rights from Mississippi to the Algebra Project. Boston: Beacon.
- Nasir, N. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking & Learning*, 4(2–3), 213–247.
- Noguera, P. (2001, September). Racial Politics and The Elusive Quest for Excellence and Equity in Education. *Motion Magazine*.
- Noss, R., Pozzi, S., & Hoyles, C. (1999). Touching epistemologies: Meanings of average and variation in nursing practice. *Education Studies in Mathematics*, 40(1), 25–51.
- Pollatsek, A., Lima, S., & Well, A. (1987). Concept of computation: Students' understanding of the mean. *Educational Studies in Mathematics*, 12(2), 191–204.
- Reveles, J. M., Cordova, R., & Kelly, G. J. (2004). Science literacy and academic identity formulation. *Journal of Research in Science Teaching*, 41(10), 1111–1144.
- Tharp, R. G., & Gallimore, R. (1989). Rousing minds to life: Teaching and learning in social context. New York: Cambridge University Press.
- Warren, B., & Rosebery, A. (1995). Equity in the future tense: Redefining relationships among teachers, students and science in linguistic minority classrooms. In W. Secada, E. Fennema, & L. Adajian (Eds.), New directions for equity in mathematics education (pp. 298–328). New York: Cambridge University Press.
- Wenger, E. (1998). Communities of Practice. New York: Cambridge University Press.
- Wortham, S. & Contreras, M. (2002). Struggling towards culturally relevant pedagogy in Latino diaspora. *Journal of Latinos in Education*, 1, 133–144.